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Bayesian Estimation of a Pragmatic Model for Monetary Policy Analysis: The Case of Pakistan

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Contents

Abstract

We present Bayesian maximum likelihood estimation of quarterly projections model for Pakistan, as documented in Ahmad & Pasha (2015). Estimation results based on quarterly data from 2001Q3 to 2023Q1 show substantial differences in values of estimated versus calibrated parameters related to aggregate demand, aggregate supply, monetary policy rule and exogenous shock processes. Comparison of pseudo out-of-sample forecasting performance for key macro variables shows that estimated model provides more precise forecasts in case of headline inflation, real GDP growth, interest rate and exchange rate over 8-quarters forecast horizon. We use the estimated model for gap analysis and scenario analysis. Gap analysis, based on March 2023 data, shows that Pakistan is passing through a recession with overshot exchange rate. In scenario analysis, we incorporate implications of political instability, climate risks, commodity prices and global financial conditions for next three years' forecasts of domestic variables under baseline and alternate scenarios. Results of scenario analysis, which may be utilized for macro stress testing exercise, show that simultaneous realization of assumed risk factors may lead to substantial deterioration of macroeconomic outlook. We explore different monetary and fiscal policy options to counter the recent crisis. Our results show that under current circumstances, using an expansionary monetary policy may lead to substantial rise in inflation and macroeconomic volatility without offering sustainable gains in GDP growth.

Keywords: Bayesian Analysis, Applied General Equilibrium Models, Forecasting and Simulation

JEL Codes: C11, D58 and E37

Non-Technical Summary

The research paper "Bayesian Estimation of a Pragmatic Model for Monetary Policy Analysis: The Case of Pakistan" aims to refine Pakistan's monetary policy analysis by enhancing an initial version of its quarterly projection model through Bayesian estimation. In line with global best practices, Bayesian estimation is the preferred approach to estimate numeric values of general equilibrium model parameters.

The flexibility of calibration approach allows to choose parameter values from multiple sources including data analysis, literature and expert opinion. At initial stage of general equilibrium modeling, this flexibility is needed as well as recommended so that the general equilibrium model could be made operational and basic model properties could be analyzed. However, once basic model properties are understood, Bayesian maximum likelihood (B-ML) estimation of the model parameters is recommended to improve data fit and forecasting performance. The B-ML estimation also incorporates calibrated parameter values in the form of prior beliefs.

The study compares forecasting performances of the estimated model versus the calibrated version, using quarterly data from 2001 to 2023. The results demonstrate the superiority of the estimated model in forecasting key macroeconomic variables like inflation, GDP growth, interest rate, and exchange rate.

Additionally, the paper utilizes the model for gap and scenario analysis, exploring monetary and fiscal policy options in response to Pakistan's economic crisis. The results of gap analysis show that Pakistan economy was passing through a recession with overshot exchange rate. The results of scenario analysis show that simultaneous realization of shocks related to political uncertainty, climate change, commodity prices and global financial conditions may lead to substantial deterioration of economic outlook. In terms of policy options, the scenario analysis shows that using accommodative policy may lead to elevated inflation without sustainable gains in growth.

1 Introduction

State Bank of Pakistan (SBP)[1] has been using a semi structural general equilibrium model for monetary policy analysis and forecasting[2]. The model, generally known as Quarterly Projections Model (QPM), was developed by (Berg et al. (2006*a*) and Berg et al. (2006*b*)). With different variations, the QPM serves as the core model used in Forecasting and Policy Analysis Systems (FPAS) that are being used by IMF desk economists and many central banks across the globe[3]. These models constitute the key analytical tool in (flexible) inflation targeting frameworks. Ahmad & Pasha (2015) customized the framework for Pakistan and provided *calibration* of long run and short run parameters in line with economic fundamentals. The authors show that the calibrated model has been capable of outperforming a suite of econometric models in terms of inflation forecasting performance during relatively calm periods. After rolling out of this initial version, SBP modeling team has been improving the model by including features like fiscal sector, public debt and further disaggregation of inflation. Accordingly, the existing version of QPM used by SBP has richer specification than presented in Ahmad & Pasha (2015).

This study offers three main contributions: (i) improvement of Pakistan's quarterly projection model by offering Bayesian maximum likelihood (B-ML) estimation of the model presented by Ahmad & Pasha (2015), (ii) establishing the superiority of B-ML estimated model through comparison of forecasting performance and, (iii) utilization of B-ML estimated model for scenario analysis to analyze Pakistan's economic crisis and discussion of potential policy options.

B-ML estimation of the model is motivated by following reasons. First, there is a need to regularly update model parameters to incorporate changes in economic relationships among domestic variables, changes in policy makers' behavior and changes in various exogenous shock processes. As the calibrated model is run using fixed parameters, it fails to incorporate changes in economic relationships and may provide misleading policy solutions. Second, while calibration approach pins down model parameters in an ad-hoc manner utilizing multiple methods, B-ML approach offers parameter estimates based on the likelihood function derived from the model and is, therefore, more suitable for the general equilibrium model. Third, alongside being a formal statistical method, B-ML is flexible enough to incorporate other-than-data information in the form of prior distributions. Fourth, the B-ML approach

[1] The central bank.

[2] See Minutes of Monetary Policy Committee Meetings available at https://www.sbp.org.pk/cmad/2020/MPC-23-Nov-2020-Eng.pdf and Ahmad et al. (2018).

[3] See, for instance Andrle et al. (2013), Charry & Thakoor (2014) and Bhattacharya & Patnaik (2014). Douglas Laxton's website provides a list of 35 countries where FPAS models have either been completed or are under construction: http://www.douglaslaxton.org/fpas.html.

is capable of addressing potential model misspecification and identification issues (An & Schorfheide (2007)).

On the basis of these strengths, B-ML estimation of the model is likely to improve forecast accuracy of the model and quality of scenario analysis conducted for inflation targeting based monetary policy analysis. In fact, our results testify that B-ML estimated model outperforms its calibrated version in a pseudo out-of-sample forecasting exercise for inflation, GDP growth, interest rate and exchange rate. Through improvement in forecasting performance of key macroeconomic variables, our paper directly contributes to macroeconomic policy analysis.

Literature also supports B-ML estimation of QPMs. Stressing the gradual nature of macroeconomic modeling process, Berg et al. (2006*b*) suggest a six-step strategy for development of QPMs. First four steps involve review of existing models and their properties to guide development of an initial calibrated version of the model. The fifth step suggests utilization of Bayesian estimation procedure to formalize the parameterization process. The last step highlights the importance of evolutionary process of macroeconomic modeling for institutional knowledge and informs that the entire process may take several years. For Pakistan, first four steps have already been completed by Ahmad & Pasha (2015) and now we are rightly positioned to move ahead by formalization of parameterization through B-ML estimation of the model. Hence, this paper bridges an important gap in the area of applied macroeconomic modeling for a developing economy.

Considering Pakistan's recent economic crisis, we utilize the model to foresee economic outcomes in baseline and an alternate (stress) scenario over the medium term. Both of the scenarios incorporate key risk factors relevant for Pakistan including political instability, climate risk, global commodity prices and global financial conditions. We also explore different monetary and fiscal policy options to achieve disinflation along with stabilizing GDP growth and exchange rate.

Rest of the paper is organized as follows: section 2 reviews literature to explore historical evolution of FPAS-type models in the context of inflation targeting, section 3 informs about the structure and equations of the model, section 4 discusses our parameterization strategy and estimation results using calibration, ML and B-ML approaches, section 5 evaluates different parameterizations through comparison of forecast performance, section 6 utilizes the model for policy analysis in three ways: (i) ceteris paribus policy analysis through impulse response functions (IRFs), (ii) gap analysis and, (iii) scenario analysis for next three years. The last section 7 concludes.

2 Literature Review

First publicly available documentation[4] of FPAS-type model was made by Czech Republic (Coats et al. (2003)). In this publication, Ales Capek & Vavra (2003) shed light on historical background in which a transition economy like Czech Republic gradually moved from fixed exchange rate regime to inflation targeting. Shift to inflation targeting necessitated availability of analytical tools that could explain the current situation of economy, have the ability to incorporate monetary policy transmission mechanism and could offer forward looking policy analysis by linking medium term forecasts of key macroeconomic variables with the current monetary policy decisions.

The Czech Republic FPAS comprised a core Quarterly Projections Model (QPM) and a set of other models that either provided inputs for the core model or were used to elaborate the output of the QPM. Polak & Vavra (2003) inform that the QPM was aided in terms of inputs by various models like autoregressive moving average with exogenous factors (ARMAX), small linear error correction models and optimizing IS curve etc. Further, satellite models were utilized to decompose the aggregate forecast output of QPM.

Beneš & Vlcek (2003) provide a detailed account of motivation, macroeconomic linkages implied by key equations, calibration and model properties for QPM. They emphasize that key purpose of the model was never to achieve accuracy in short term forecasting of inflation or other macroeconomic variables as sector specialists and judgment based forecasts were capable of providing these short term forecasts. Rather, the key objective was to build a framework that could provide theoretically consistent evaluation of risks to baseline forecast in the medium term. Apart from being easy to communicate, the framework should be able to guide the policy makers what needed to be done to achieve inflation target in the medium term.

The QPM was designed to be a gap model showing how economy converges to steady state as policy makers try to achieve inflation target in the medium term. Including definitions and definition type relationships, the model had around 85 equations. However, key equations including aggregate demand, monetary policy rule, modified uncovered interest rate parity condition, Phillips curve, Okun's law and equations for term as well as risk premia were not more than 14. Owing to lack of data under market economy setup, calibration method was preferred over formal statistical estimation to find numeric values of the model parameters. The authors demonstrate that the framework was capable of providing a theoretically plausible impact assessment for a range of shocks to economy e.g. shocks to inflation target, aggregate demand, cost push inflation and exchange rate etc.

[4] To the best of our knowledge.

Berg et al. (2006*a*) introduce the structural model used by IMF economists for monetary analysis for several countries. They describe the model as a convergence between micro-founded DSGE models and empirically motivated IS/LM models. They explain that more than being a forecasting device, the model should be viewed as a tool to analyze issues related to monetary policy. They discuss advantages of the model over alternative approaches used for monetary policy analysis including Financial Programming Framework and econometric models like VAR, SVAR and VECM.

The authors explain that Financial Programming has limited capability of conducting monetary policy owing to instability of money demand function and lack of ability to answer forward looking monetary policy related questions. On the other hand, econometric models like VAR, SVAR and VECM suffer from following issues. First, sufficient macro data required to estimate these models might not be available due to unavailability of time series or regime shifts. Second, these models generally show very weak monetary policy transmission and large changes in monetary policy tools are therefore required to attain desired policy goals. Third, these models suffer from Lucas Critique and are not fit for forward looking policy analysis. Fourth, estimated reduced form models cannot be used to study changes in assumptions regarding economic environment e.g. changes in exchange rate pass through etc. Finally, identification issues related to SVAR make these models less useful for monetary policy analysis.

In contrast to these limitations, the authors show that their proposed structural model is consistent with world view of inflation targeting monetary policy makers where central role of monetary policy is to anchor inflation expectations. Rationale of adopted monetary policy stance is better communicated through a relatively small and easy to understand model.

Berg et al. (2006*b*) provide the technical details of the model discussed above in general and parameterization strategy in particular. The model mainly describes four key relationships including dynamic IS curve (aggregate demand equation), Phillips curve, modified uncovered interest rate parity condition and central bank' reaction function. The authors note the then recent advancements in DSGE modeling and Bayesian estimation methodology and provide a detailed six-steps strategy that chalks out transition from calibrated small macro models to more sophisticated models parameterized under Bayesian estimation approach. Key steps of their strategy involve (1) review of existing models' structure, (2) analysis of existing models' properties to guide about calibration of new model, (3) development of an initial model, (4) comparison of new model's properties with different models, (5) Bayesian estimation and (6) promotion of institutional knowledge. The authors note that this process generally takes many years. Finally, the authors present an application of the model for Canadian economy.

With an objective to support flexible inflation forecast targeting regime (IFT), Peiris et al. (2011) provide application of a model similar to Berg et al. (2006*b*) for the case of Sri Lanka. They estimate the model using Bayesian ML method using data from 1996Q1 to 2010Q3. Priors for Bayesian estimation were based on calibrated parameters from existing studies. The authors show that forecasting performance of the model was better than Bayesian VAR model for forecast horizon beyond two to three quarters. They work out optimal weights of response to inflation, output gap and exchange rate in the central bank's reaction function and show that existing estimated weight for exchange rate was larger than optimal weight; indicating that caring too much about stabilizing exchange rate may lead to increase in overall macroeconomic volatility.

Since low income countries are generally vulnerable to aggregate supply shocks in general and food inflation shocks in particular, Andrle et al. (2013) extend the core macro model used in FPAS to specifically model the behavior of food and non-food inflation rates. After explicit incorporation of these supply shocks in the form of separate Phillips curves, the model was better suited for monetary policy analysis that actually involves aggregate demand rather than aggregate supply management. The authors present the application of their model in case of Kenya. They parameterize the model using calibration approach and present impulse response functions, shock decompositions, Kalman filter based trend-gap decomposition and forecasts.

Dizioli & Schmittmann (2015) present application of FPAS framework provided in Berg et al. (2006*b*) for the case of Vietnam. Using quarterly data from 2000 to 2014, they estimate the model using Bayesian ML method and perform impulse response analysis together with in-sample as well as out of sample forecasting. They also work out optimal weights for monetary policy rule.

Apart from many other countries, FPAS has been utilized for monetary policy analysis in India as well. There are, at least, two applications of FPAS available for India. First contribution was made by Bhattacharya & Patnaik (2014) in which calibrated version of Berg et al. (2006*b*) model was applied to Indian case. Kalman filter based trend-gap decomposition and shock decomposition were obtained for quarterly data from 1996 to 2013. Impulse response analysis was also provided. The second contribution was made by (Benes et al. 2017) wherein they provide a multi sector calibrated Quarterly Projection Model to support flexible inflation targeting (FIT) regime by the central bank. Owing to high share of food in Indian CPI (46 percent) and prominence of food inflation shocks, the model has separate Phillips curves for food, core and energy inflation rates. The model also incorporates bank lending channel. The authors study model properties using impulse response analysis and present shock decompositions to work out a model-based description of historic macroeconomic events.

To sum up, Pakistan is gradually moving towards flexible inflation targeting [5]. For successful implementation of this regime, the central bank needs a structural macroeconomic model to evaluate different policy trade-offs and preparation of medium term baseline forecasts together with risk analysis associated with the baseline. Further, the model should be easy to communicate and capable of linking today's monetary policy actions with future trajectories of macroeconomic outcomes in general and inflation in particular. As discussed at length in this section, many central banks across the globe including developed and developing economies are using QPMs under FPAS-type setups for this purpose. In case of Pakistan, although we have a QPM which is being utilized by FPAS (Ahmad & Pasha (2015)), yet the model needs to be improved through Bayesian estimation; as recommended by the six-step strategy discussed in Berg et al. (2006*b*). The current paper contributes to academic as well as policy oriented macroeconomic literature by improving the model through Bayesian estimation approach.

3 Model

Since equations of the model have already been presented and interpreted in Ahmad & Pasha (2015), therefore we avoid duplication and briefly discuss the structure of the model. The model has in total 58 equations including 21 structural equations, 4 trend equations, 20 identities definitions and 13 measurement equations. The model is divided into four blocks: (i) aggregate demand; (ii) aggregate supply; (iii) external sector; (iv) and policymakers' forward-looking interest rate rule. We briefly explain the key macroeconomic linkages in these four sectors. Equations of the model are presented in Appendix A. For detailed information about the model, please see Ahmad & Pasha (2015).

3.1 Aggregate Demand

The aggregate demand equation shows that output gap ($\hat{y}_t$) is a function of lagged output gap ($\hat{y}_{t-1}$), monetary conditions index (mci_t) and foreign output gap ($\hat{y}^*_t$). Monetary conditions index mci_t is a weighted average of real interest rate gap and real exchange rate gap [6]. An increase in mci_t is associated with tightening of monetary conditions and vice versa. mci_t may increase due to increase in real interest rate gap or due to real appreciation of domestic

[5] https://www.sbp.org.pk/70/sup-12.asp

[6] Nominal exchange rate S_t is defined as local currency units per US dollar. Real exchange rate z_t is defined as $z_t = S_t \frac{CPI_t^{US}}{CPI_t^{PK}}$.

currency (associated with decrease in real exchange rate) and vice versa. In absence of explicit modeling fiscal block, fiscal policy shocks may also be accommodated under a shock to aggregate demand.

3.2 Aggregate Supply

The multi-sector aggregate supply block contains equations on the lines of New Keynesian Phillips curves (NKPCs) for core, food and energy inflation. These NKPCs contain lagged impact of quarter-on-quarter (QoQ) inflation, a forward looking component based on headline inflation and sector-specific marginal costs. In terms of explicit micro foundations, the autoregressive lagged component could be obtained by augmenting pure forward looking NKPC with complete indexation of prices as in Christiano et al. (2005) or partial indexation of prices as in Smets & Wouters (2003) and Smets & Wouters (2007). The marginal costs include local as well as imported components through variations in exchange rate, world oil prices and world food prices. The QoQ headline inflation is obtained by weighted average of core, food and energy inflation rates. QoQ inflation rates have been converted into year-on-year (YoY) through different identities.

3.3 External Sector

This model uses an augmented version of uncovered interest parity (UIP) condition. In this modified UIP condition, exchange rate expectations have forward looking as well as backward looking components. Backward looking component of exchange rate expectations depends upon inertia and fundamentals implied by purchasing power parity (PPP) theory.

3.4 Policy-Maker Behavior

The central bank's interest rate rule responds to external and domestic factors that may affect inflation outcomes. External factors include changes in expected depreciation of local currency, foreign interest rate and risk premium. Domestic factors include interest rate smoothing, deviation of four-quarters ahead inflation forecast from inflation target and output gap. Weight of external factors is g_1 whereas weight of domestic factors is $1 - g_1$.

In Ahmad & Pasha (2015), central bank responded to expected change in depreciation rate of domestic currency:

$$
\begin{aligned}
i_t &= g_1\left[\frac{1}{1-e_1}E_t\Delta s_{t+1} - \frac{e_1}{1-e_1}\Delta s_t + i_t^* + prem_t\right] + \\
&\quad (1-g_1)\left[f_1 i_{t-1} + (1-f_1)\left\{i_t^n + f_2\left(E_t\pi_{t+4} - \pi_t^T\right) + f_3\hat{y}_t\right\}\right] + \varepsilon_t^i \\
&= g_1\left[(E_t\Delta s_{t+1} - \Delta s_t) + i_t^* + prem_t\right] + \\
&\quad (1-g_1)\left[f_1 i_{t-1} + (1-f_1)\left\{i_t^n + f_2\left(E_t\pi_{t+4} - \pi_t^T\right) + f_3\hat{y}_t\right\}\right] + \varepsilon_t^i
\end{aligned}
$$

To make central bank's response to currency depreciation more transparent, we replace expected change in depreciation rate by depreciation rate:

$$
\begin{aligned}
i_t &= g_1\left[E_t\Delta s_{t+1} + i_t^* + prem_t\right] + \\
&\quad (1-g_1)\left[f_1 i_{t-1} + (1-f_1)\left\{i_t^n + f_2\left(E_t\pi_{t+4} - \pi_t^T\right) + f_3\hat{y}_t\right\}\right] + \varepsilon_t^i
\end{aligned}
$$

4 Parameterization

The model has in total 59 parameters. These parameters may be divided into two broad groups: the long run (or balanced growth path, 6 parameters) parameters and the business cycle parameters.

The business cycle parameters can be grouped as the parameters governing relationships among different variables (20 parameters), the parameters governing the shock processes (32 parameters) and measurement errors (1 parameter). There are two broad ways to parameterize the DSGE models: calibration and estimation. The estimation approach can be further classified into maximum likelihood (ML) estimation and Bayesian maximum likelihood (B-ML) estimation.

4.1 Calibration

Calibration involves measurement of model parameters through information from micro economic studies, matching the important macro ratios like consumption to GDP, $\frac{c}{y}$ and investment to GDP, $\frac{i}{y}$ etc. in data and model, matching the cyclical properties of the model with the observed data, taking parameters values from other studies and using judgment in the form of policy makers' inputs or most educated guess (Heijdra & Ploeg (2002)). Key strength of calibration approach is its flexibility in the sense that it does not preclude model simulation only because parameter values do not conform to some established econometric criteria. The most important limitation of calibration approach is lack of a coherent method while choosing a broad range of parameters.

Table 1: Steady State Parameters

Parameter	**Symbol**	**Calibration**
Steady state domestic inflation	$\bar{\pi}$	9.00
Steady state foreign inflation	$\bar{\pi}^*$	2.00
Steady state real interest rate	$\bar{rr}$	0.62
Steady state foreign real interest rate	$\bar{rr}^*$	1.30
Steady state growth of real exchange rate	$\bar{z}$	-0.10
Steady state real GDP growth rate	$\bar{y}$	4.20

Following Ahmad & Pasha (2015), we calibrate long run trend parameters using sample averages of relevant data series. Estimates of long run parameters are given in Table 1. Comparing the values of long run parameters with Ahmad & Pasha (2015), a few notable differences can be observed. First, long run average of domestic inflation; used as long run inflation target in the model, has increased from 8% to 9%.[7] Second, growth rate of real exchange rate has increased from -1.5% to -0.10 per cent. Finally, domestic real interest rate average has decreased from 1.0% to 0.62%. These variations are consistent with the high inflation and depreciation trends observed in the recent past. Apart from long run trend parameters, coefficients of weight of food, w_{food} and energy inflation, w_{oil} are also calibrated to be 0.35 and 0.10, respectively.

4.2 Maximum Likelihood Estimation

If DGP[8] is given by $y = x\beta + \epsilon$ where y is $T \times 1$ vector of dependent variable, x is $T \times k$ matrix representing T observations of k independent variables and, $\epsilon \sim N(0, \sigma^2)$, then, PDF for for t^{th} observation would be:

$$p(y_t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y_t - x_t\beta)^2}{2\sigma^2}\right]$$

[7] The inflation target quoted in SBP monetary policy statements is 5-7%. In policy simulations, using a higher number for inflation target may imply a weaker monetary policy response. On the other hand, during in-sample model evaluation, using a lower number for average inflation may imply a poor forecasting performance.

[8] Data generating process.

Assuming that all observations are independent, joint probability for entire vector y, or likelihood function is given by multiplication rule:

$$\begin{aligned} p(y_1, y_2, ...y_T) &= p(y_1).p(y_2)....p(y_T) \\ L &= \prod_{t=1}^{T} \left(2\pi\sigma^2\right)^{-\frac{1}{2}} \exp\left[-\frac{\left(y_t - x_t\beta\right)^2}{2\sigma^2}\right] \\ &= \left(2\pi\sigma^2\right)^{-\frac{T}{2}} \prod_{t=1}^{T} \exp\left[-\frac{\left(y_t - x_t\beta\right)^2}{2\sigma^2}\right] \end{aligned}$$

where $\theta = [\beta, \sigma^2]$ is the vector of parameters. Likelihood function is generally expressed in natural log terms to facilitate differentiation:

$$\ln L = -\frac{T}{2}\left(\ln 2\pi\right) - \frac{T}{2}\left(\ln \sigma^2\right) - \frac{1}{2\sigma^2}\left(y - x\beta\right)^{'}\left(y - x\beta\right)$$

As in this case of linear regression model with normality assumption, log-likelihood can be maximized with respect to parameters β and σ^2 to work out ML estimators.

However, ML based parameterization has also some issues. First, likelihood function can be extremely complex and non-linear function of parameters. Second, likelihood function can be very flat and optimization could be difficult. Finally, likelihood function can provide parameter estimates which are against economic logic.

We tried to estimate the business cycle parameters through ML approach. However, owing to the issues mentioned in the previous para, only a subset of parameters could be estimated satisfactorily. Results of ML estimation are provided in Table 9 and, Figure 18 & Figure 19 in Appendix B.

4.3 Bayesian Maximum Likelihood Estimation

Bayesian estimation fundamentally differs from classical (or frequentist) approach: frequentists consider data as random outcome of DGP governed by fixed parameters. On the other hand Bayesians consider observed data as fixed while parameters are treated as random variables. While frequentist approach solely relies upon data for inference, Bayesians utilize information from data as well as information from other than data sources in the form of priors to sharpen the inference.

Use of prior information in the form of prior distributions ensures that estimated parameter values lie in sensible range [9]. Hence, B-ML serves to be the logical middle path between

[9] For instance, we do not want estimated standard deviation of shocks to be negative.

highly subjective calibration approach and purely data-driven ML approach. The strengths and flexibility offered by Bayesian analysis effectively overcome the parameterization issues encountered in calibration and ML approaches. Accordingly, most of the modern studies using DSGE models generally use Bayesian maximum likelihood estimation methods to parameterize models (Smets & Wouters (2003), Smets & Wouters (2007) and Adolfson et al. (2007)). Practically, Bayesian analysis is still aided by calibration where B-MLE is not able to identify the parameter values.

Under B-ML, posterior distributions of parameters are estimated conditional upon observed data. Coefficients that are used in model simulations are obtained by central tendency measures (mean, median or mode) of posterior distribution. Using Bayes law, posterior distribution can be worked out as:

$$p(\theta|y) = \frac{p(\theta)p\left(y|\theta\right)}{p(y)} \tag{1}$$

$$= \frac{p(\theta)p\left(y|\theta\right)}{\int p(y|\theta)p(\theta)d\theta} \tag{2}$$

where $p(\theta|y)$ denotes posterior distribution of estimated parameters conditional upon observed data set y. Prior distribution of parameters is denoted by $p(\theta)$ while $p\left(y|\theta\right)$ and $p(y) = \int p(y|\theta)p(\theta)d\theta$ represent likelihood function (or data density) and marginal distribution of data, respectively. Since marginal distribution is a fixed constant, therefore we may note that posterior is proportional to prior and likelihood:

$$p(\theta|y) \propto p(\theta)p\left(y|\theta\right) \tag{3}$$

Variations in posterior distribution depend upon variations in prior distributions and likelihood function. In this context, we may note that effectively Bayesian analysis allows researchers to update their prior beliefs under the light of information available in data (the likelihood function).

Against this backdrop, choice of prior distributions is an important step of Bayesian analysis. Following literature on DSGE models, we choose Normal distribution for parameters that could be either negative or positive, β distribution for parameter that are confined in $(0, 1)$ interval and γ^{-1} distribution for standard deviation of shocks that are constrained to be positive. Means and initial values of prior distributions are based upon calibration of Ahmad & Pasha (2015).

As shown above, Bayesian analysis yields posterior distribution rather than point estimates of parameters. To find point estimates, Bayesian risk or posterior expected loss function is minimized:

$$\arg\min_{\hat{\theta}} E\left(L(\theta,\hat{\theta})|y\right) = \arg\min_{\hat{\theta}} \int L(\theta,\hat{\theta})p(\theta|y)d\theta$$

Minimization of quadratic loss function yields posterior mean, minimization of absolute error loss function yields posterior median and $0-1$ loss function minimization provides posterior mode. To assess uncertainty around point estimates, highest posterior density (HPD) regions may be constructed. Bayesian statistics does not have any absolute measure of data fit like R^2. However, relative posterior support of different models may be compared through posterior odds ratio, $PO_{i,j}$.

$$PO_{i,j} = \frac{p(M_i|y)}{p(M_j|y)} = \frac{p(M_i)p(y|M_i)}{p(M_j)p(y|M_j)}$$

If priors are same, then Bayes' factor $BF_{i,j}$ can be used to compare two models:

$$BF_{i,j} = \frac{p(y|M_i)}{p(y|M_j)}$$

Bayesian analysis requires integration of posterior density function to obtain parameter values. For likelihood function of DSGE models, generally closed form analytical solutions are not available and we have to use numerical simulation methods to generate simulated posterior distributions. To this end, Markov Chain Monte Carlo (MCMC) methods are used. In these methods, two widely used methods are Gibbs Sampler and Metropolis-Hastings (MH) algorithm. We have utilized MH algorithm implemented in macroeconomic modeling toolbox IRIS.

4.4 State Space Representation of the Model

State space representation of rational expectations solution of DSGE model is given as:

$$X_t = RX_{t-1} + S\epsilon_t \tag{4}$$

where X_t is a vector of state variables of the model. Since all of the variables are not observable (output gap, exchange rate gap and marginal costs etc.), therefore measurement equation relates model variables with actual data.

$$y_t^{obs} = \Gamma + VX_t + e_t \tag{5}$$

where y_t^{obs} is vector of observable variables, Γ is vector of constants and V is the matrix selecting and linking state and observable variables.

Model variables and observed data are linked through measurement equations. Allowance for measurement errors is important in general and for Pakistan macro data in particular. Unavailability of quarterly national income accounts data necessitates the use of quarterly interpolated series of GDP. We use quarterly real GDP data based on Tahir et al. (2018) and introduce measurement error for quarterly real GDP to incorporate the uncertainty regarding quarterly interpolation. Existence of measurement errors is likely to insulate estimations results from unavailability of quarterly national income accounts data.

4.5 Discussion of Estimation Results

Results of B-ML are presented in Table 2 along with calibrated values of parameters to facilitate comparison. During comparison of calibrated versus estimated parameters, we focus upon the cases where substantial differences between the two types of parameters are observed. Comparison of aggregate demand parameters shows that weight of real interest rate in monetary conditions index (MCI), a_4 is 0.72, considerably higher than the calibrated number of 0.20. Aggregate supply parameters show that core inflation turns out to be slightly more persistent as estimated parameter b_1 is 0.76 compared with 0.67 calibrated value. Further, core inflation appears to be more sensitive to changes in real marginal costs (RMC) as estimated sensitivity coefficient b_2 is 0.38 which is substantially higher than calibrated value of 0.15.

Estimation results show that sensitivity of food inflation towards food-specific real marginal costs, b_{22} is 0.45, lower than calibrated value of 0.70. However, share of external factors b_{23}, represented by relative food price is estimated to be 0.85, which is considerably higher than calibrated value of 0.25. This shows that external factors i.e. world food prices and exchange rate are more important than considered by previous calibration. This finding is consistent with the recurrent import requirement of key food items like wheat and sugar due to domestic shortage. Oil prices pass through from global to domestic energy prices, b_{32} has been estimated to be 0.20, which is lower than calibrated value of 0.70.

Regarding external sector, share of backward looking expectations in modified UIP, e_1 is estimated to be 0.44. Although lower than calibrated value of 0.60 yet, the number is close to 50%, showing relevance of backward looking expectations. Monetary policy reaction towards expected depreciation, g_1 is estimated to 0.09 which is substantially lower than calibrated value of 0.60. This finding is consistent with the adoption of market based exchange rate after start of IMF Extended Facility (EFF) in July 2019.

Comparison of Taylor-type interest rate rule shows that interest rate smoothing coefficient, f_1 is 0.87, which is substantially higher than calibrated value of 0.60. Previous calibration assumed that central bank assigned more importance to inflation in comparison with growth concerns, as indicated by calibrated values of $f_2 = 0.80$ and $f_3 = 0.20$. However, estimation results show that estimated magnitudes for responses to both expected inflation and output gap are almost equal i.e. $f_2 = 1.84$ and $f_3 = 1.83$.

Persistence of shock to FX risk premium $\rho^{\epsilon^{prem}}$ is estimated to be 0.77, substantially higher than the calibrated value of 0.50. The calibrated model applied several restrictions by setting various shock persistence parameters equal to each other. For instance, h_1, simultaneously representing persistence of shock to growth of trend real exchange rate $\rho^{\epsilon^{\bar{\dot{z}}}}$, trend real interest rate $\rho^{\epsilon^{\bar{rr}}}$, trend foreign real interest rate $\rho^{\epsilon^{\bar{rr}^*}}$ and trend real GDP growth $\rho^{\epsilon^{\bar{\dot{y}}}}$, was calibrated to be 0.50. However, in unconstrained model, estimated values of these coefficients are found different from each other and larger than calibrated value of 0.50. Similarly, persistence of shocks to world food inflation $\rho^{\epsilon^{\pi^{wfood}}}$ and world oil prices inflation $\rho^{\epsilon^{\pi^{woil}}}$ was calibrated at $t_1 = 0.10$. However, estimated values of these parameters are 0.45 and 0.30, substantially higher than 0.10.

Standard deviations (SDs) of shocks play an important role in model dynamics. For instance, SDs of gaps are higher than corresponding trend variables. Similarly, variations among SDs for different shock processes dictate relative importance of different shocks in terms of explaining observed data. Estimated values of SDs for shocks to domestic energy inflation $\sigma^{\pi^{oil}}$, world oil inflation $\sigma^{\pi^{woil}}$, world food inflation $\sigma^{\pi^{wfood}}$ and growth of real exchange rate trend $\sigma^{\bar{\dot{z}}}$ are substantially higher than the corresponding calibrated values. On the other hand, SDs of shocks to output gap $\sigma^{\hat{y}}$, nominal exchange rate σ^{s}, monetary policy shock σ^{rn}, foreign output gap $\sigma^{\hat{y^*}}$, foreign inflation σ^{π^*}, foreign nominal interest rate σ^{rn^*} and risk premium shock σ^{prem} are considerably lower than corresponding calibrated values. Estimated value of measurement error for quarterly real GDP, $\sigma^{y_{me}}$ is 0.06 which is slightly lower than calibrated value of 0.10.

In sum, B-ML estimates reveal significant differences from their calibrated counter parts, justifying the use of B-ML to update parameters before using model for policy analysis on quarterly basis. These differences may be attributable to difference in estimation methodology and/or difference in estimation sample. In any case, using B-ML procedure provides a systematic way to update parameters.

Table 2: Business Cycle Parameters

Parameter	Symbol	Calibrated	Estimated Posterior Mode
Aggregate Demand			
Output gap persistence	a_1	0.60	0.57
Impact of MCI on output gap	a_2	0.10	0.12
Impact of foreign GDP on output gap	a_3	0.15	0.12
Weight of real interest rate in MCI	a_4	0.20	0.72
Persistence in credit premium	a_5	0.80	0.50
Aggregate Supply			
Core inflation persistence	b_1	0.67	0.76
Impact of RMC on core inflation	b_2	0.15	0.38
Weight of output gap in RMC	b_3	0.85	0.70
Food inflation persistence	b_{21}	0.30	0.25
Impact of food RMC on food inflation	b_{22}	0.70	0.45
Weight of relative food price in food RMC	b_{23}	0.25	0.85
Energy inflation persistence	b_{31}	0.03	0.05
Impact of energy RMC on domestic energy inflation	b_{32}	0.70	0.20
Weight of food in CPI basket	w_{food}	0.35	
Weight of energy in CPI basket	w_{oil}	0.10	
External Sector and Monetary Policy			
Weight of backward-looking elements in UIP	e_1	0.60	0.44
Interest rate smoothing	f_1	0.60	0.87
Interest rate reaction to inflation forecast deviation	f_2	0.80	1.84
Interest rate reaction to output gap	f_3	0.20	1.83
Interest rate reaction to expected depreciation	g_1	0.60	0.09
Persistence of Shock Processes			
Persistence of shock to FX risk premium	$\rho^{\epsilon^{prem}}$	0.50	0.77
Persistence of shock to trend growth of real ER	$\rho^{\epsilon^{\dot{\bar{z}}}}$	0.50	0.74
Persistence of shock to trend real interest rate	$\rho^{\epsilon^{r\bar{r}}}$	0.50	0.68
Persistence of shock to trend real GDP growth rate	$\rho^{\epsilon^{\dot{\bar{y}}}}$	0.50	0.54
Persistence of shock to foreign real interest rate	$\rho^{\epsilon^{r\bar{r}*}}$	0.50	0.68
Persistence of shock to foreign GDP gap	$\rho^{\epsilon^{\hat{y*}}}$	0.50	0.67
Persistence of shock to foreign interest rate	$\rho^{\epsilon^{rn*}}$	0.50	0.91
Persistence of shock to foreign inflation	$\rho^{\epsilon^{\pi*}}$	0.50	0.41
Persistence of shock to world food inflation	$\rho^{\epsilon^{\pi^{wfood}}}$	0.10	0.45
Persistence of shock to world oil inflation	$\rho^{\epsilon^{\pi^{woil}}}$	0.10	0.30
Persistence of shock to target nominal ER growth	$\rho^{\epsilon^{s\dot{t}ar}}$	0.50	0.57
Persistence of shock to target inflation	$\rho^{\epsilon^{\pi^{tar}}}$	0.50	0.41
Standard Deviations of Shock Processes			
Std dev of shock to output gap	$\sigma^{\hat{y}}$	1.00	0.22
Std dev of shock to headline inflation	σ^{π}	0.60	0.26
Std dev of shock to core inflation	$\sigma^{\pi^{core}}$	0.50	0.44
Std dev of shock to energy inflation	$\sigma^{\pi^{oil}}$	3.00	4.77
Std dev of shock to food inflation	$\sigma^{\pi^{food}}$	0.70	1.35
Std dev of shock to nominal ER	σ^{s}	0.70	0.44
Std dev of shock to monetary policy rate	σ^{rn}	1.00	0.26
Std dev of shock to growth of target ER	$\sigma^{s\dot{t}ar}$	0.50	0.69
Std dev of shock to target inflation	σ^{target}	0.50	1.13
Std dev of shock to foreign GDP	$\sigma^{\hat{y*}}$	0.50	0.19
Std dev of shock to foreign interest rate	σ^{rn*}	1.00	0.12
Std dev of shock to foreign equilibrium real interest rate	$\sigma^{r\bar{r}*}$	0.50	0.49
Std dev of shock to foreign inflation	$\sigma^{\pi*}$	0.50	0.71
Std dev of shock to trend real interest rate	$\sigma^{r\bar{r}}$	0.50	0.56
Std dev of shock to trend growth of real ER	$\sigma^{\dot{\bar{z}}}$	0.50	1.29
Std dev of shock to trend real GDP growth	$\sigma^{\dot{\bar{y}}}$	0.40	0.76
Std dev of shock to FX risk premium	σ^{prem}	0.70	0.82
Std dev of shock to food relative price	$\sigma^{z\hat{food}}$	0.80	0.76
Std dev of shock to world food price	$\sigma^{\pi^{wfood}}$	1.00	5.00
Std dev of shock to world oil inflation	$\sigma^{\pi^{woil}}$	6.00	15.00
Standard Deviation of Measurement Error			
Std dev of shock to measurement error in GDP	σ^{yme}	0.10	0.06

5 Evaluation of the Model

As discussed in literature review section 2, major objective of QPM is to facilitate forward looking policy analysis by linking current policy decisions with the future economic outcomes. When the general equilibrium characteristic of QPM is augmented with comparable forecasting performance, we have a policy tool which is convincingly useful for forward looking policy analysis. In this context, medium term forecasting performance becomes an important evaluation criterion for the model. Although many empirical models like ARIMA and (Bayesian) VAR are used for forecasting yet, they have limited applicability in terms of forward looking policy analysis as they lack theoretical foundations and general equilibrium structure. For a detailed comparison of QPM versus competing forecasting approaches, please see Berg et al. (2006*a*).

The initial calibrated version of the model was evaluated through comparison of forecast performance for only headline inflation by Ahmad & Pasha (2015). The authors showed that the model was capable of outperforming a suite of econometric models in relatively tranquil periods. We extend the range of variables and evaluate the model through forecasting performance comparison of headline, core and food inflation rates, real GDP growth rate, exchange rate and interest rate. As objective of our paper is to improve an existing semi structural model rather than choice between some empirical versus semi structural model, therefore we do not include empirical models in forecast comparison and focus upon the three parameterizations of the model (i.e. B-ML, calibration and ML).

5.1 Pseudo Out-of-Sample Recursive Forecasting

We recursively generate pseudo out-of-sample forecasts of above mentioned variables for 8-quarters forecast windows. We have in total 46 windows of 8-quarters forecast horizon with 2010Q-2011Q4 and 2021Q2 - 2023Q1 being the first and the last forecast windows, respectively. Figure 1 provides display of recursive forecasting for B-ML specification. The bold black line shows actual values while thin red lines show recursive forecasts. During this exercise, external variables like world oil prices, world food prices, USA inflation, GDP and interest rate are assumed equal to their actual values[10].

Forecast errors and root mean squared errors (RMSEs) are computed by comparing forecasts and actual variables for respective quarters. Table 3 and Figure 2 summarize the

[10]Being a small open economy model, the framework focuses upon domestic variables and uses simple autoregressive equations for most of the external variables. Therefore, during real-time policy analysis, it is advisable to use consensus forecast of external variables rather forcing the model to provide forecast of these variables. For scenario analysis in section 6, we also use World Economic Outlook (WEO) and Global Projections Model (GPM) forecasts for baseline.

information available in Figure 1 for the three parameterizations by providing RMSEs. Lower RMSE indicates more precise forecast. Results show that B-ML estimated model clearly outperforms the calibrated counterpart in case of headline inflation, real GDP growth rate and interest rate. In case of exchange rate too, B-ML model wins over calibrated model; albeit with a thin margin. In case of core inflation, B-ML outperforms calibration but is beaten by ML. For food inflation, B-ML and calibrated models show similar performance which is better than ML.

Overall results of forecast performance show that B-ML provides the most suitable forecasts relative to competing parameterization strategies in case of headline inflation, GDP growth, interest rate and exchange rate. The largest gains in forecast performance are observed in cases of real GDP growth rate and interest rate. These results justify the use of B-ML procedure every quarter before using the model for forecasting and policy analysis.

Table 3: Root Mean Squared Errors from Expanding Window Recursive Forecasting

	Parameterization	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
CPI Inflation QoQ	Calib	4.3	4.2	4.6	4.5	5.0	6.2	6.5	7.7
	ML	3.8	3.6	3.9	3.9	4.4	5.8	5.9	7.1
	B-ML	3.9	3.6	4.0	4.0	4.4	5.8	5.9	7.1
CPI Inflation YoY	Calib	1.1	1.6	2.2	2.5	2.6	3.3	4.0	4.9
	ML	1.0	1.5	2.0	2.4	2.5	3.2	3.8	4.7
	B-ML	1.0	1.5	2.0	2.4	2.4	3.2	3.7	4.6
Core Inflation QoQ	Calib	2.3	2.9	3.5	4.1	4.6	5.2	5.8	6.0
	ML	1.7	1.8	2.0	2.3	2.6	3.1	3.6	3.7
	B-ML	2.0	2.2	2.6	3.2	3.7	4.3	4.7	4.9
Food Inflation QoQ	Calib	6.5	6.4	6.4	6.3	6.8	7.8	8.2	10.0
	ML	7.0	6.9	7.1	7.1	7.5	8.3	8.6	10.4
	B-ML	6.7	6.4	6.3	6.3	6.8	7.7	8.1	10.0
Exchange Rate, PKR/USD	Calib	4.1	5.9	7.5	9.1	11.6	15.3	17.9	24.3
	ML	5.1	8.4	11.4	14.2	17.4	21.2	24.1	29.9
	B-ML	4.2	5.9	7.4	8.9	11.3	14.7	17.3	23.6
Nominal Interest Rate	Calib	1.7	2.6	3.1	3.4	3.6	3.7	3.7	3.7
	ML	2.1	2.9	3.3	3.6	3.8	3.8	3.9	3.9
	B-ML	0.9	1.5	2.0	2.4	2.8	3.0	3.2	3.4
Real GDP Growth YoY	Calib	1.4	2.0	2.3	2.4	2.0	2.1	2.3	2.5
	ML	1.6	2.4	2.7	2.9	2.3	2.3	2.5	2.8
	B-ML	1.1	1.4	1.5	1.6	1.7	1.9	2.1	2.3

Figure 1: Recursive Forecast Charts for B-ML Parameterization

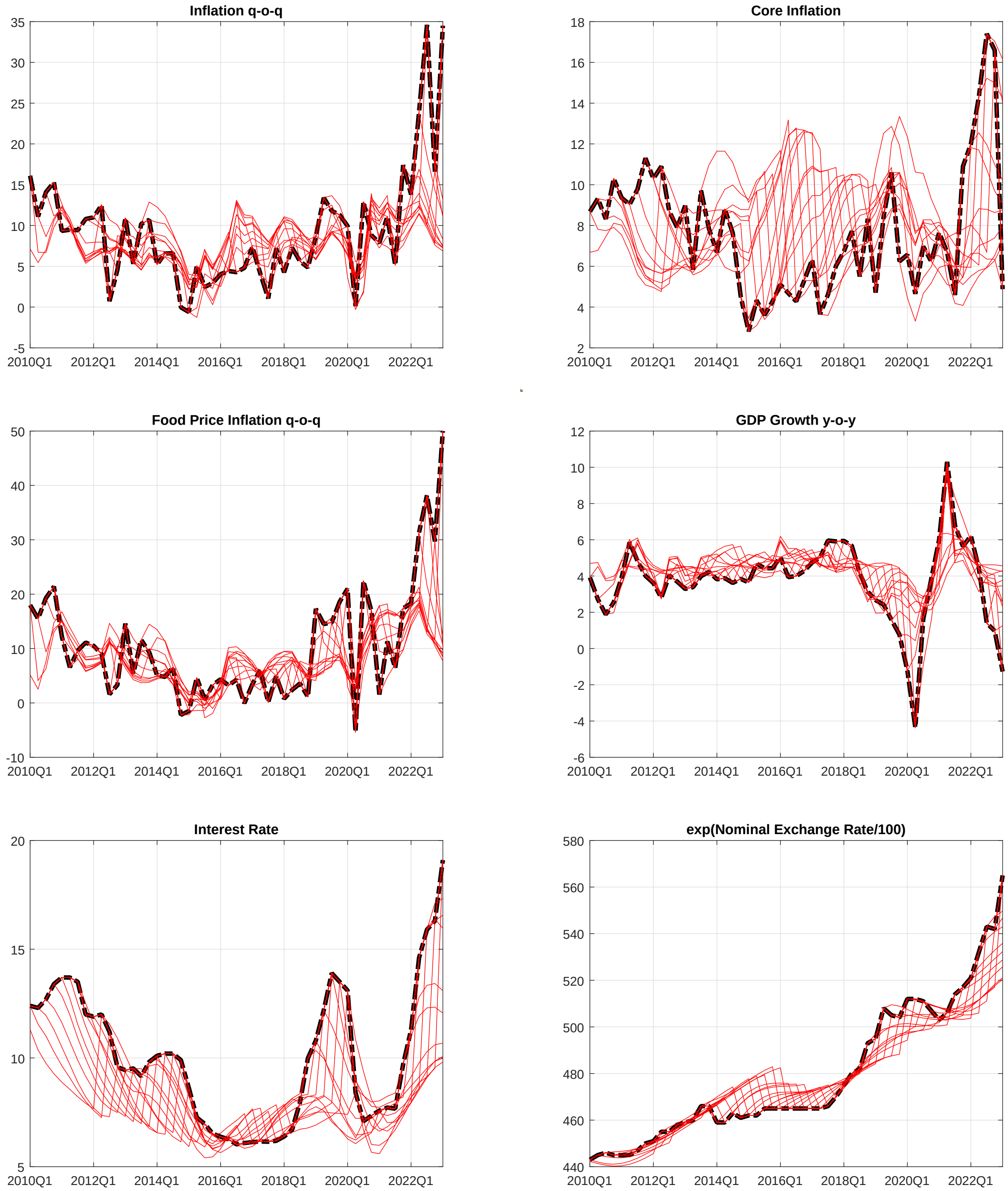
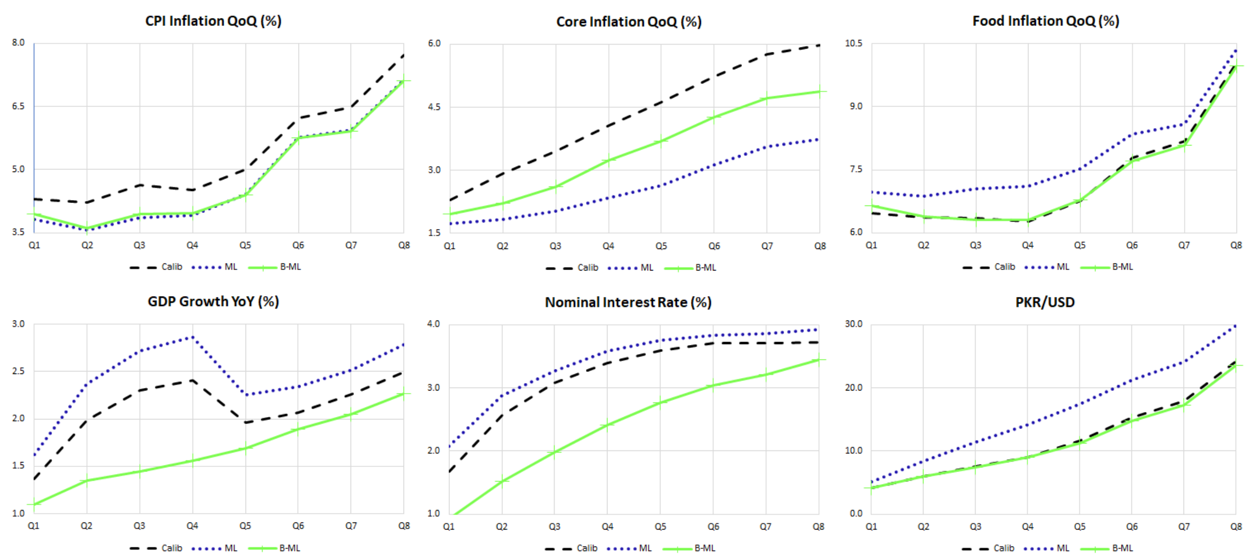


Figure 2: Root Mean Squared Errors of Forecasts



6 Use of the Model for Policy Analysis

In order to use the model for policy analysis, we focus on three applications: gap analysis, impulse response functions and scenario analysis.

6.1 Gap Analysis

We use the model through multivariate Kalman filter to link the observed data with latent variables like long term trends and cyclical components. In log-linearized form, variables of micro-founded models are gaps or deviations from their respective steady states. Being an equilibrium model, all variables converge to their respective steady state values or long run trends in the long run. Therefore, identified magnitudes and signs of gaps have important implications for forecasts.

Apart from implications for forecasting exercise, gaps are direct policy inputs. We focus upon the three most important gap indicators namely output (or GDP) gap $\hat{y}_t$, real exchange rate gap $\hat{z}_t$ and real interest rate gap $\hat{r}_t$. Weighted average of $\hat{r}_t$ and $\hat{z}_t$ constitute monetary conditions index mci_t. Weighted average of $\hat{y}_t$ and $\hat{z}_t$ constitute real marginal costs for core inflation rmc_t. Since mci_t and rmc_t respectively affect aggregate demand and aggregate supply, therefor these variables play a critical role in determination of monetary policy response and policy transmission mechanism.

Positive output gap implies that economy is operating beyond its productive capacity, leading to upward pressure on factor prices that ultimately leads to pressure on inflation. Real exchange rate gap measures exchange rate misalignment. Positive exchange rate gap implies that domestic currency is under valued relative to foreign currency. Under valuation of domestic currency boosts exports and discourages imports. On the other hand, negative exchange rate gap implies real appreciation that hampers export competitiveness, leading to decline in aggregate demand. Positive real interest rate gap implies that monetary policy stance for domestic borrowers is relatively tight and vice versa.

Figure 3 shows the actual, trend and gap values in case of real GDP, exchange rate and interest rate. At the end of 2023Q1, economy was in recession as shown by output gap. Domestic currency was under valued, as shown by positive real exchange rate gap. Real interest rate was negative owing to high inflation.

6.2 Impulse Response Functions

Impulse response functions (IRFs) show the responses of the model's endogenous variables to a hypothetical shock in a single variable. In most cases, we have assumed 1% impulse in shocked variables. Our IRFs are obtained under two key assumptions. First, the shocks to structural variables are assumed to be orthogonal. Second, only the impulse variable is assumed to change according to the assumed magnitude of shock while all other variables remain unchanged at their steady state. Hence, these IRFs are consistent with assumptions of ceteris paribus analysis. The horizontal and vertical axes on charts of IRFs respectively show number of quarters and deviation from steady state level. To illustrate transmission mechanism of different shocks, we focus upon responses of nine endogenous variables including headline, core and food inflation rates; interest rate, exchange rate depreciation, output gap, real interest rate gap, real exchange rate gap and monetary conditions index (MCI).

6.2.1 Nominal Interest Rate Shock

Figure 4 shows the impact of 1% shock to monetary policy rate on key macroeconomic variables under the parameterizations of calibration, ML and B-ML. Increase in interest rate leads to tight monetary conditions through increase in real interest rate and real appreciation of domestic currency. Tighter monetary conditions reduce output gap and real marginal costs, easing pressure on core inflation. Being the major part of CPI basket—around 55%, reduction in core inflation leads to fall in headline inflation. Through expectations' channel, this reduction in headline inflation lowers food and oil inflation rates as well. The results show that B-ML parameterization is consistent with the maximum impact of monetary policy shock on inflation.

6.2.2 Aggregate Demand Shock

Figure 5 shows the impact of aggregate demand shock on key macroeconomic variables under the three parameterizations. Positive shock in output gap (interpreted as increase in aggregate demand) leads to an increase in core inflation via increase in real marginal costs (RMC). Rise in core inflation contributes to upward pressure on headline inflation. Central bank responds to expected rise in headline inflation and increase in output gap by increasing interest rate. Increase in nominal interest rate leads to appreciation of real exchange rate and increase in real interest rate; culminating into tightening of MCI. As a result, aggregate demand reverts back towards steady state level, rmc_t declines and inflationary pressure subsides. Results show that ML parameterization is consistent with the maximum impact of AD shock on macro variables under discussion.

Figure 3: Gap Analysis using Multivariate Kalman Filter

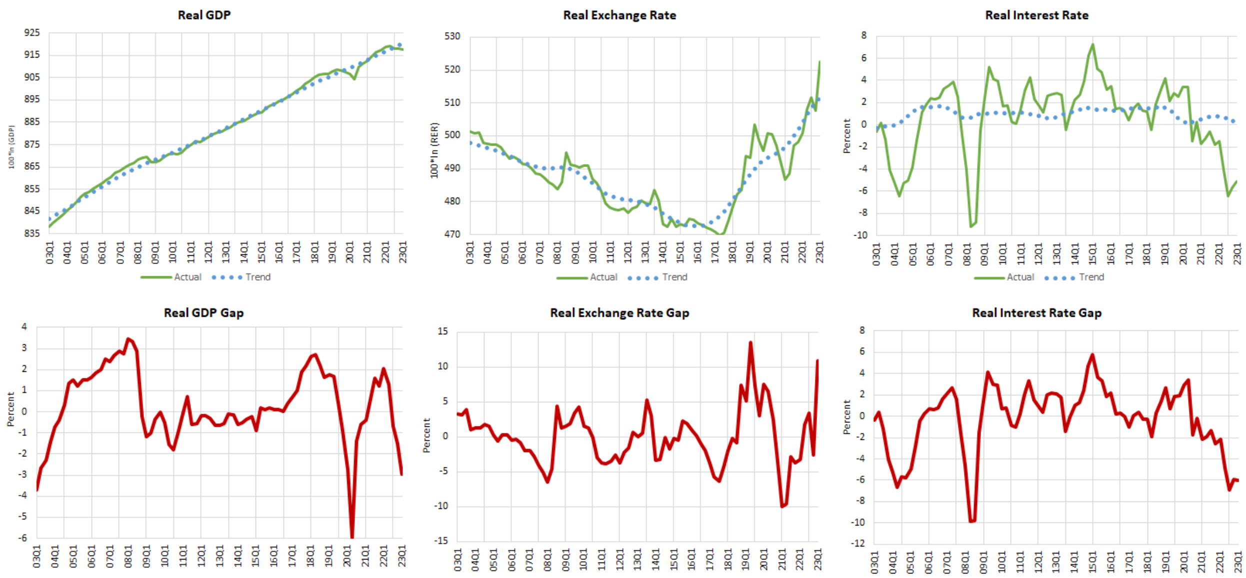


Figure 4: Impulse Response Functions for Monetary Policy Shock

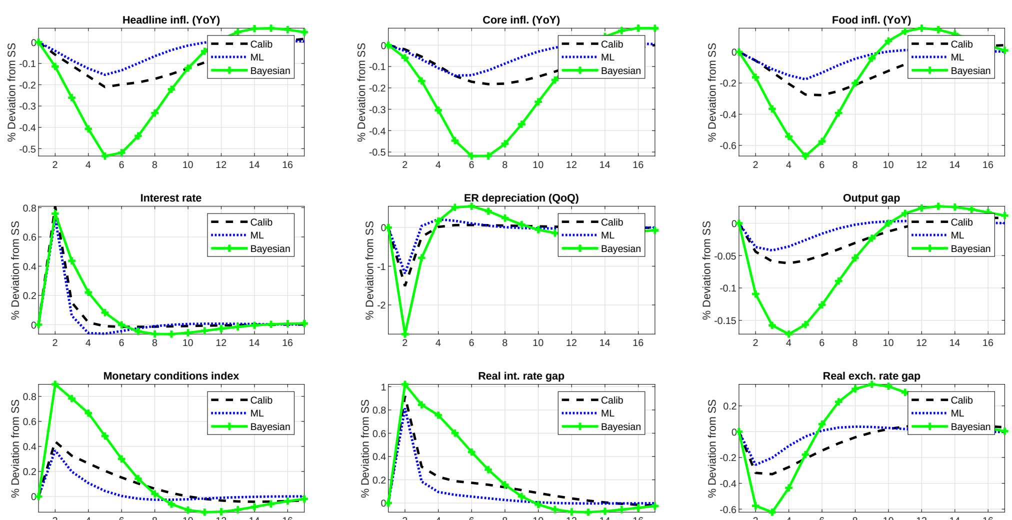
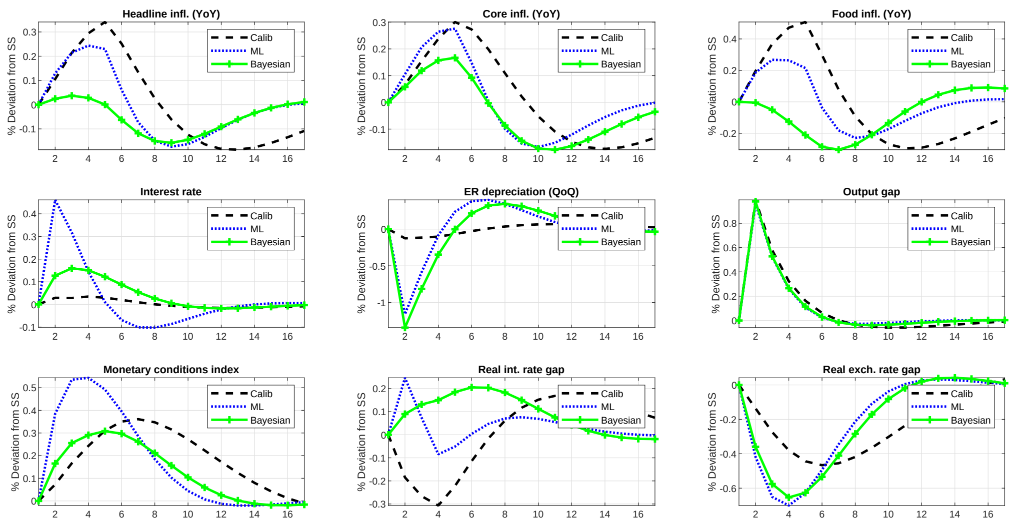


Figure 5: Impulse Response Functions for Domestic Demand Shock



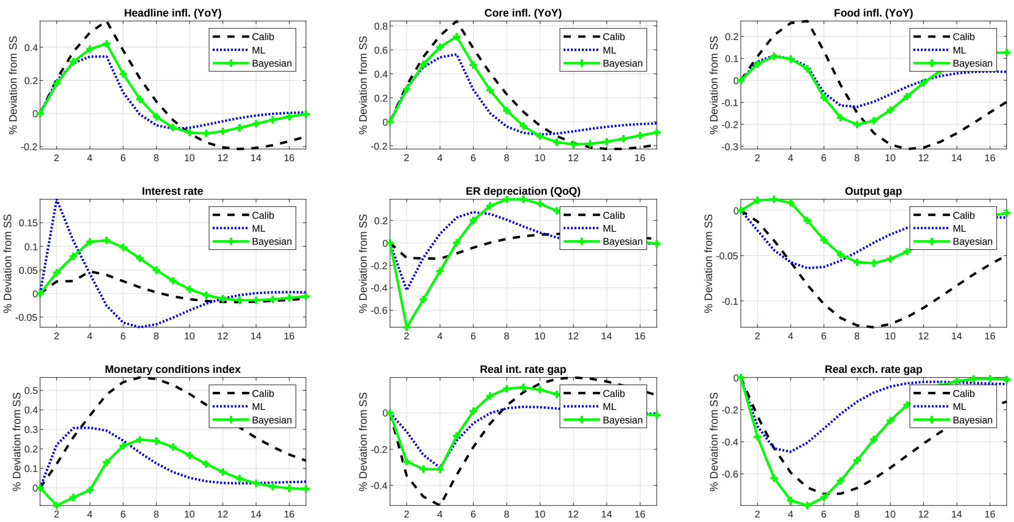
6.2.3 Core Inflation Shock

Figure 6 shows the impact of 1% shock to QoQ core inflation on key macroeconomic variables. YoY core inflation gradually rises in response to shock in QoQ inflation, leading to rise in headline inflation which attracts central bank's policy response in the form of increase in interest rate. Increase in interest rate leads to appreciation of local currency and leads to negative real exchange rate gap. Real interest rate gap, after initial dip due to rise in inflation, also becomes positive after six quarters due to rise in monetary policy rate. Hence real interest rate gap and real exchange rate gap both contribute to tightening of MCI which in turn negatively affects output gap. Reduction in output gap and downward movement in real exchange rate gap lower RMC, leading to ease in core inflation.

6.2.4 Risk Premium Shock

Figure 7 shows the impact of shock to country risk premium on key domestic macroeconomic variables. Increase in risk premium leads to a sharp depreciation of domestic currency that affects RMC and gradually pushes up YoY rates of core, food and headline inflation. Rise in expected inflation and domestic depreciation make central bank respond by raising interest rate. MCI shows an initial short lived tightening spike and then reflects a persistent dip of negative movement indicating easing in monetary conditions. These variations in MCI can be explained by sharp rise in interest rate followed by gradual increase in headline inflation. In response to these variations, output gap shows initial decline followed by a persistent

Figure 6: Impulse Response Functions for Core Inflation Shock



hump. After six to seven quarters, MCI turns slightly positive with corresponding negative movement in output gap. All domestic indicators come back to steady state level in 16 quarters.

6.2.5 World Oil Price Shock

Figure 8 shows the impact of 1% increase in world oil prices on key domestic macroeconomic variables. Increase in world oil prices passes through to domestic energy inflation via energy-specific real marginal costs. Consistent with the 10% share of energy in CPI basket, a one percent increase in oil prices leads to around 0.1 percent gradual increase in YoY headline inflation. Increase in headline inflation lowers real interest rate gap and causes a short lived ease in monetary conditions. Aggregate demand increases in response to ease in mci_t. As central bank responds to rising inflation and aggregate demand, nominal interest rate rises and mci_t increases. Tightening of mci_t leads to reversal of aggregate demand and inflation.

6.2.6 World Food Price Shock

Figure 9 shows the impact of 1% world food price shock on key domestic macroeconomic variables. Shock to world food inflation affects domestic food inflation through increase in relative food price that pushes up food-specific real marginal costs. Increase in domestic food inflation pushes up headline inflation; leading to negative interest rate gap. Central bank responds to increase in inflation by increasing monetary policy rate. Increase in interest rate

Figure 7: Impulse Response Functions for FX Risk Premium Shock

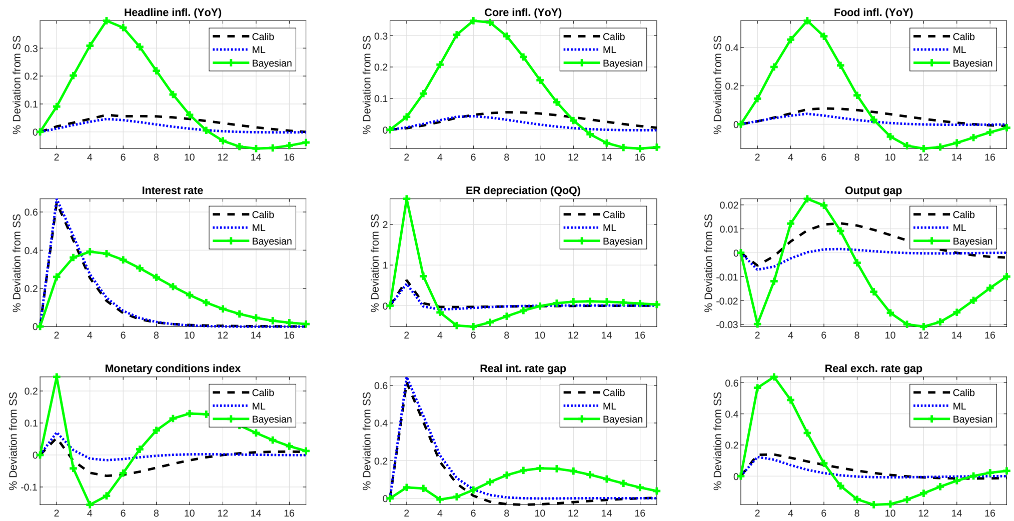


Figure 8: Impulse Response Functions for World Oil Prices Shock

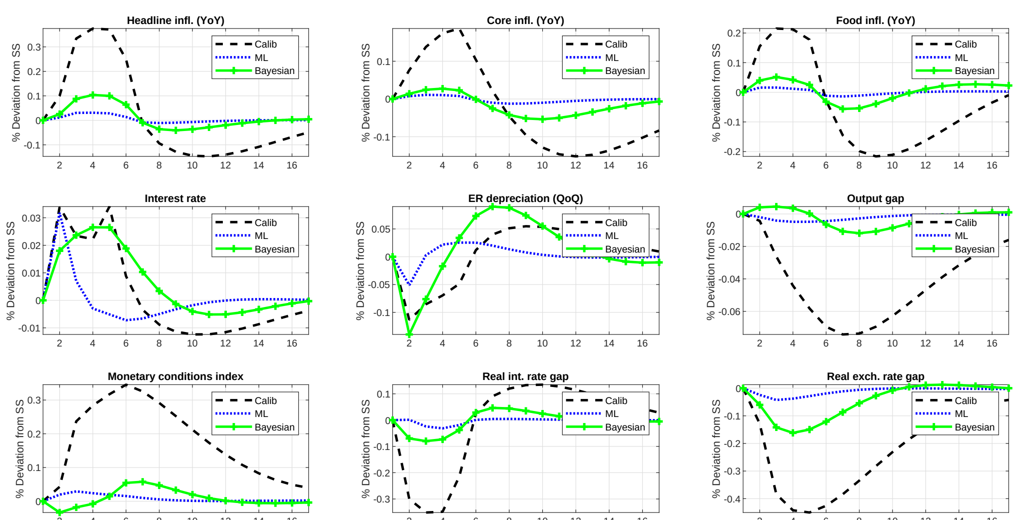
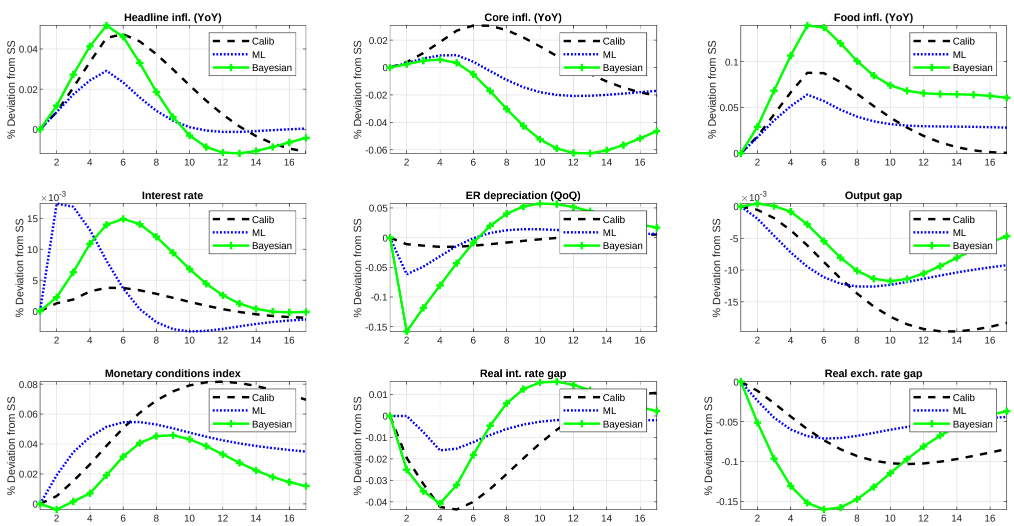


Figure 9: Impulse Response Functions for World Food Prices Shock.



leads to appreciation of local currency. Although interest rate gap indicates ease in MCI yet real exchange rate gap points to the converse. Overall impact is tightening of MCI that reduces output gap and eases inflationary pressure.

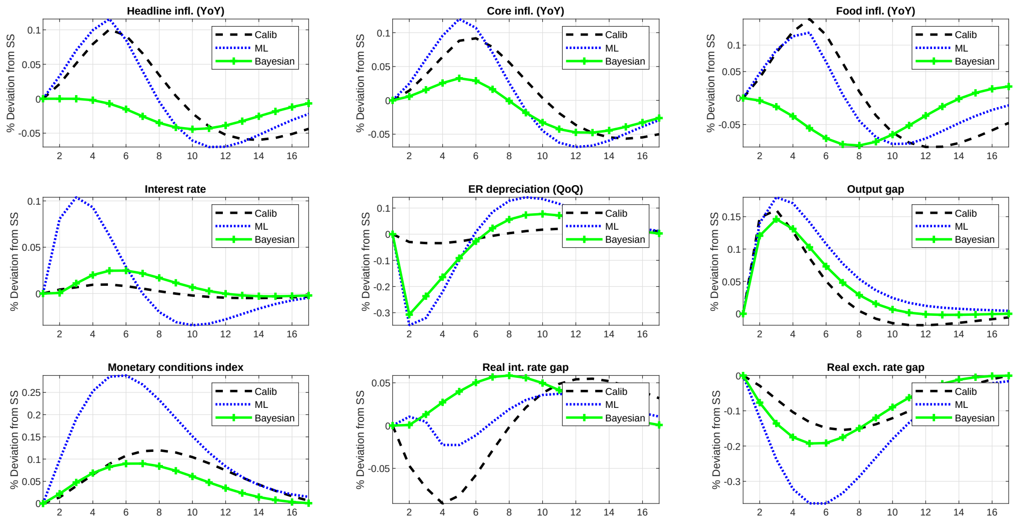
6.2.7 Foreign Demand Shock

Figure 10 shows response of domestic variables towards a 1% impulse in foreign (export) demand shock. Increase in foreign demand directly affects domestic aggregate demand, as shown by aggregate demand equation. Increase in output gap leads to rise in RMC and increase in core inflation. Since output gap is one of the arguments of central bank's monetary policy rule, therefore monetary policy rate is increased. Increase in interest rate leads to exchange rate appreciation and tightening of MCI. Tightening of MCI reduces domestic output gap, RMC and eases pressure on inflation.

6.2.8 Global Financial Conditions Shock

Figure 11 shows the impact of impulse in world interest rate, the key indicator of global financial conditions, on key domestic macroeconomic indicators. Increase in world interest rate leads to sharp and persistent depreciation of domestic currency, as modeled by modified UIP condition. The depreciation leads to positive real exchange rate gap, pushes RMCs for core and food NKPCs and causes a gradual surge in corresponding YoY inflation rates. Resultantly, headline inflation rises. Since depreciation and expected inflation are two argu-

Figure 10: Impulse Response Functions for Foreign Demand Shock



ments of central bank's policy rule, therefore central bank responds through raising the policy rate. A persistent rise in interest rate eliminates initial easing in MCI, leads to tightening of monetary conditions and eases pressure on aggregate demand and inflation.

6.2.9 Foreign Inflation Shock

Figure 12 shows the impact of impulse in foreign inflation on domestic macroeconomic indicators. Increase in foreign inflation leads to appreciation of domestic currency through modified UIP condition. However, as global central banks increase foreign interest rates to counter inflationary pressures, domestic currency comes under pressure and appreciating trend gets reversed. The pressure on domestic currency is indicated by positive real exchange rate gap that leads to decrease in RMC and increase in core as well as food inflation rates. To overcome the pressure on domestic currency, domestic central bank raises policy rate. Increase in interest rate leads to increase in real interest rate gap which eliminates the slack in MCI and brings inflation and output gap back to steady state levels.

6.3 Forecasting and Scenario Analysis

Medium term forecasting is the most important application of QPM type models. Forecasting through a structural model like QPM provides far richer economic analysis than mechanical prediction of economic variables. Key objective of forecasting exercise is to offer a forward looking economic analysis of whole macroeconomic system while understanding the

Figure 11: Impulse Response Functions for World Interest Rate Shock

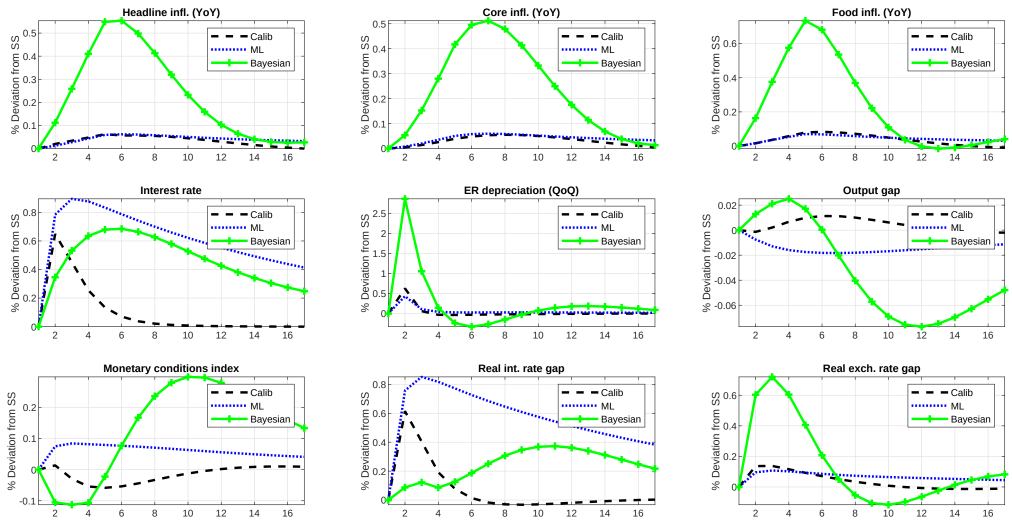
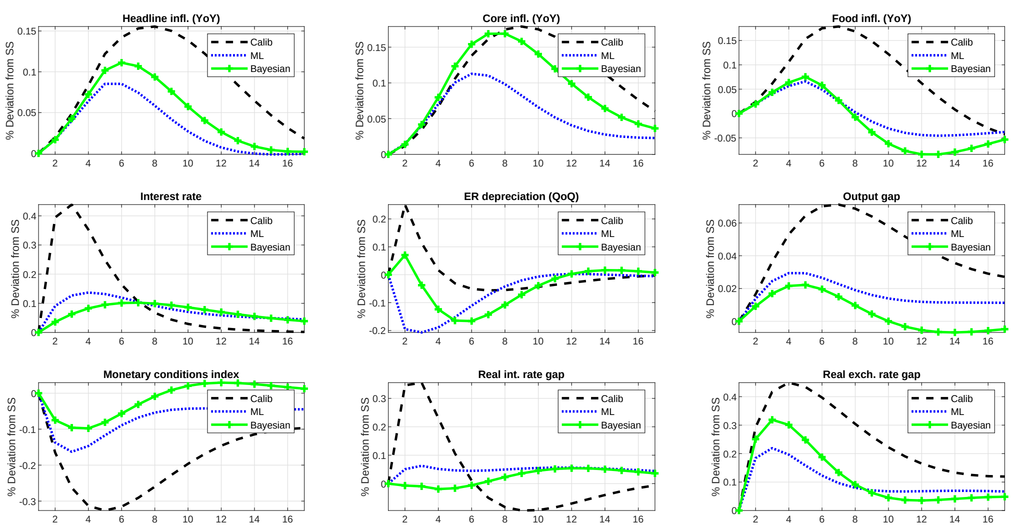


Figure 12: Impulse Response Functions for World Inflation Shock



linkages between potential monetary policy decisions and the key macroeconomic indicators like inflation, GDP growth and exchange rate.

Apart from monetary policy analysis, QPM projections may also be utilized in Macro Stress Testing (MST) exercise. In MST, we map stress scenario projections of macroeconomic variables on balance sheet of financial institutions to see whether they remain resilient and solvent during macroeconomic stress. For meaningful MST exercise, macroeconomic shocks should fulfill a few basic conditions. First, the shocks should be extreme enough to replicate a truly stressed macroeconomic environment. Second, the shocks should not be too extreme to be plausible. Finally, the stressed scenarios should ensure theoretical consistency for assumed trajectories of variables. The stressed scenarios generated through estimated QPM fulfill these conditions. Stressed scenario projections from estimated QPM may be fed into satellite credit risk and market risk models to generate solvency projections of banks under the hypothetical stressed scenario.

Based on our results from section 5, we use B-ML specification for forecasting in the scenario analysis exercise. Baseline scenario provides the most probable outlook of macro variable under going conditions. Uncertainty around baseline scenario is analyzed through construction of an alternate (stress) scenario; incorporating risks to baseline forecast. Along with assessment of uncertainty around baseline, the alternate scenario results may be used for MST by feeding into appropriate satellite models developed for forecasting of credit and market risks.

6.3.1 Scenario Design

At the very outset of scenario design, it is important to clarify that the alternate scenario is based on hypothetical assumptions regarding extremely stressed conditions of global and domestic indicators. Therefore, projections under the alternate scenario should not be interpreted as forecasts.

Our scenario design incorporates the major risks faced by Pakistan economy, as documented by Financial Stability Review of SBP[11]. We consider four key risk factors including ongoing political uncertainty, climate risks, global commodity prices and tightening of world financial conditions. All of these risk factors are included in the both baseline and alternate scenarios. However, level of severity for assumed shocks varies in both the scenarios. In following lines, we discuss the transmission mechanism of assumed risk factors. In line with

[11] Financial Stability Department in SBP publishes stress testing results based on hypothetical stress scenario: https://www.sbp.org.pk/FSR/2022/Chp-4.pdf. Our alternate scenario only shares the risk factors with the hypothetical stress scenario; paths of exogenous external variables and shocks to endogenous domestic variables are provided in Table 4 and Table 5, respectively.

our understanding regarding transmission mechanism, we work out quantitative assumptions regarding baseline and alternate scenario.

Political Uncertainty Pakistan has been facing substantial amount of political uncertainty over recent past. The uncertainty has affected economy in several ways. First, the lack of a stable government has so far precluded meaningful implementation of economic reforms in fiscal sector and governance of state owned enterprises (SOEs). In the long term, lack of reforms affects economy through loss of economic efficiency and recurrence of twin deficit crises. In the short run, inability to implement the reforms continues to be an important factor behind disruptions in multilateral financing [12]. Second, political instability leads to discontinuity of economic policies and discourages long term investments in the economy. Finally, political instability leads to loss of consumer confidence, leading to reduction in aggregate demand and output.

We model implications of political instability through following variables. First, we assume that political uncertainty and associated disruptions in multilateral financing may lead to a rise in country risk premium. This assumption is consistent with the recent trend of spread in credit default swap (Figure 14). Second, political uncertainty may lead to a negative shock to consumer confidence that will reduce aggregate demand. For numeric values of assumed shocks to risk premium and aggregate demand, please refer to Table 5.

Climate Risks Notwithstanding very low share in global carbon emissions, Pakistan ranks high among the countries that are most vulnerable to the catastrophic effects of global warming and climate change. Pakistan is ranked as the 8^{th} most affected country according to the long term climate risk (CRI) index published by the Germanwatch[13]. While climate related events may affect economy in several ways like floods, droughts and heat waves etc., in most situations, they are likely to push food inflation up by damaging agriculture. On the other hand, losses to cash crops e.g. cotton may lead to pressure on external sector owing to increased demand for imported raw materials[14] along with losses to exports.

Both the baseline and alternate scenarios assume that food inflation and exchange rate will receive an upward impulse during third and fourth quarters of calendar years, potentially attributable to floods that generally hit the economy in monsoons. Climate change may lead to fluctuations in pattern and intensity of monsoon rains. However, assumed shocks in

[12] Pakistan got External Fund Facility (EFF) from IMF in July 2019. However, the program has faced several disruptions. https://www.dawn.com/news/1747931 and is being followed by a Stand-by Arrangement (SBA) for nine months.

[13] The ranking is based on data from 2000 till 2019; omitting the massive flash floods that hit Pakistan in 2022. Global Climate Risk Index 2021, https://www.germanwatch.org/en/cri

[14] For instance, damage to cotton crop will lead to rise in cotton imports by local textile industry.

alternate scenario are larger in magnitude in comparison with the baseline scenario. For precise magnitude of assumed shocks, please refer to Table 5.

Global Commodity Prices Historically, Pakistan has been a net importer of oil. In recent years, lack of timely investments in agricultural research, negative implications of climate change and issues in commodity management have been necessitating import of key agricultural items like wheat, sugar and cotton. This situation indicates towards sensitivity of Pakistan economy to fluctuations in global commodity markets.

Global commodity markets continue to be volatile. In recent past, oil prices observed historic low during Covid when average spot oil prices fell to 21.04 \$/b in April 2020[15]. However, a sharp and prolonged surge in oil prices was observed due to post Covid recovery. The surge in oil prices was further augmented by Ukraine-Russia war in 2022; pushing up the global oil prices to 116.70 \$/b in June 2022. However, as China and India keep on bypassing sanctions on Russian oil amid weak global demand attributed to slowing global economy, oil prices are gradually declining since June 2022 and quarterly average for 2023Q1 was recorded to be 79.04 \$/b.

In line with WEO April 2023 projections,[16] the baseline scenario assumes that declining trend in oil prices will continue. The alternate scenario assumes that owing to intensification of Ukraine-Russia war and stronger recovery in China, oil prices may remain at elevated level relative to the level assumed in baseline. Baseline scenario assumes that with slight downward movement, global food prices are expected to remain broadly stable over the projections horizon. However, considering the recent surge in food prices after war between Ukraine and Russia; the two major exporters of wheat, alternate scenario assumes that intensification of war may lead to unanticipated closure of Black Sea corridor that may push up world food prices[17].

Baseline and alternate scenario assumptions regarding world oil and food prices are fed into the model simulations by fixing the corresponding QoQ inflation series paths over the projections horizon. For details, please refer to Figure 13 and Table 4.

Tightening of World Financial Conditions The central banks across the globe; barring a few notable exceptions like China and India, have been responding to elevated levels of inflation through higher monetary policy rates. Although declining oil prices and contrac-

[15] Monthly average of average of Brent, Dubai & West Texas Intermediate (WTI)

[16] According to IMF World Economic Outlook, April 2023, oil prices are projected to gradually decline to 71 \$/b by 2027.

[17] In November 2022, renewal of Black Sea corridor allowed entry of Ukrainian wheat in global markets and eased pressure on wheat prices.

tionary monetary policy stance have helped ease pressure on headline inflation yet , tight conditions in labor market suggest that persistence of core inflation may lead to continuity and intensity of tight monetary policy stance (IMF WEO, April 2023). Being a small open economy with already stressed external account, tightening of global financial conditions may lead to pressure on exchange rate for Pakistan.

Baseline scenario assumes that ongoing deceleration in global inflation will continue, allowing central banks to reduce interest rate. This will boost global aggregate demand. However, the alternate scenario assumes that deceleration in foreign inflation will be smaller than baseline, forcing the central banks to keep interest rates at elevated levels for longer. This will lead to reduction in output gap relative to baseline Figure 13 and Table 4.

After comparison of baseline versus alternate scenarios, we construct different scenarios of monetary and fiscal policies to analyze available policy options to counter the recent economic crisis. We explore different monetary and fiscal policy scenarios to assess their implications for inflation, GDP growth and exchange rate.

Monetary Policy Experiments We assess two monetary policy scenarios relative to baseline: an accommodating monetary policy stance versus an extinguishing monetary policy stance. Accommodating monetary policy scenario assumes that the central bank keeps monetary policy rate two percentage points lower than the baseline path for the next four quarters. This scenario is likely to boost output growth; albeit at the cost of higher inflation. On the other hand, the extinguishing scenario assumes that the central bank keeps monetary policy rate two percentage points higher than the baseline path for the next four quarters. This scenario is likely to achieve disinflation at relatively higher pace; albeit at the sacrifice of GDP growth.

Monetary and Fiscal Policy Mix Experiments Although our framework does not explicitly models fiscal sector yet, a shock to output gap, broadly interpreted as a shock to aggregate demand, can be used to assess the implications of fiscal impulse. We augment the accommodative and extinguishing monetary policy scenarios explained above with a contractionary fiscal policy shock, represented by a negative two percent impulse to output gap for next four quarters. As a result of this policy mix, our accommodative scenario is now less expansionary and extinguishing scenario is more contractionary.

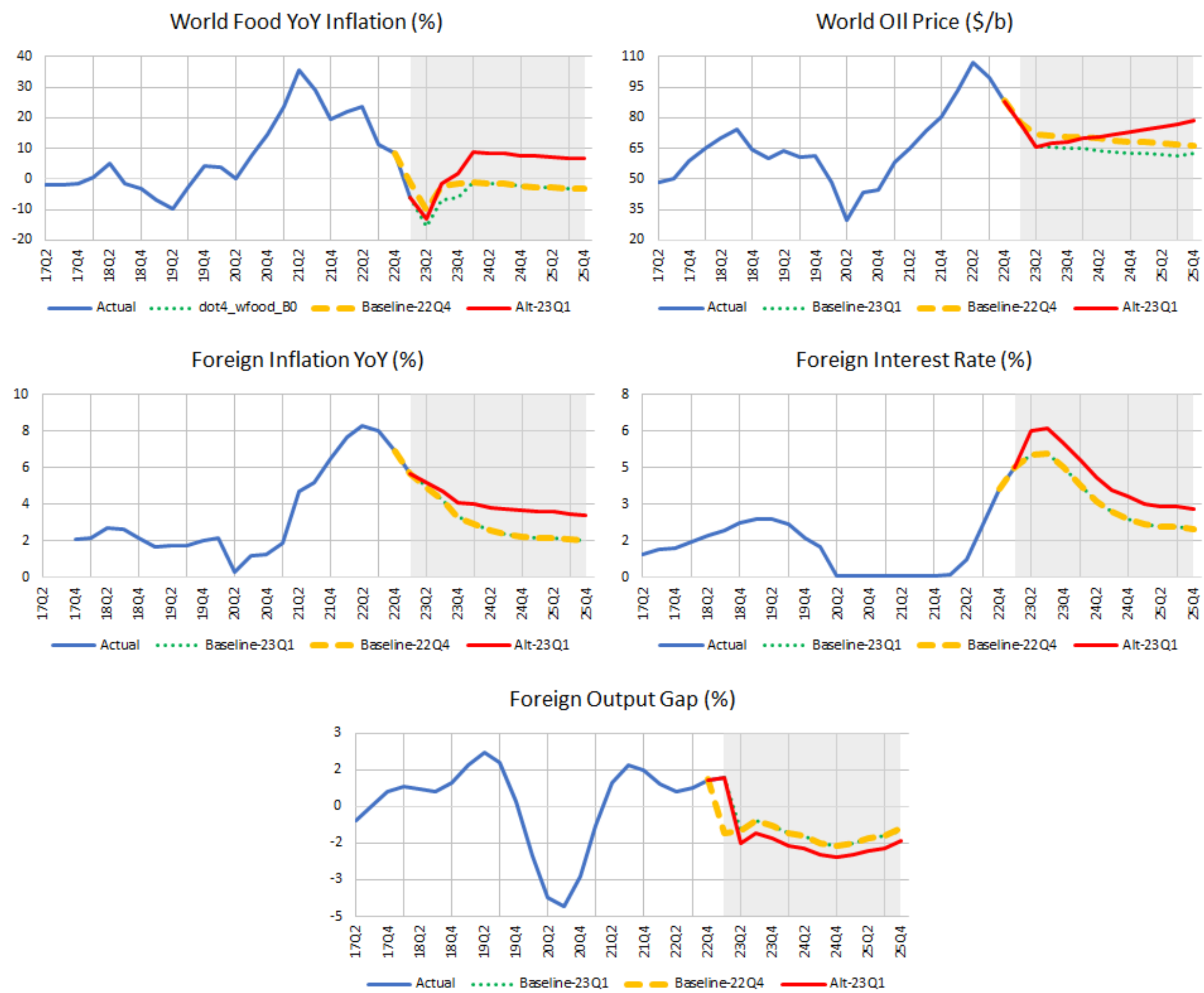
Table 4: External Environment Assumptions for Baseline and Alternate Scenarios

Variable	Scenario	23Q2	23Q3	23Q4	24Q1	24Q2	24Q3	24Q4	25Q1	25Q2	25Q3	25Q4
USA Output Gap	Baseline	-1.0	-0.6	-0.8	-1.1	-1.2	-1.5	-1.6	-1.5	-1.3	-1.2	-0.9
	Alternate	-1.5	-1.1	-1.3	-1.6	-1.7	-2.0	-2.1	-2.0	-1.8	-1.7	-1.4
USA Inflation	Baseline	6.0	2.9	0.0	2.8	4.6	2.0	-0.5	2.6	4.6	1.7	-0.7
	Alternate	7.0	3.9	1.2	4.1	6.1	3.5	0.9	4.0	6.0	3.1	0.6
USA Interest Rate	Baseline	5.0	5.1	4.5	3.8	3.1	2.7	2.4	2.2	2.1	2.1	2.0
	Alternate	6.0	6.1	5.5	4.8	4.1	3.6	3.3	3.0	2.9	2.9	2.8
World Oil Price	Baseline	79.0	78.0	78.0	77.0	76.0	75.5	75.0	74.5	74.0	73.5	73.0
	Alternate	81.0	82.0	84.0	85.0	86.0	87.6	89.2	90.9	92.5	94.2	95.9
World Food Price Index	Baseline	135.5	135.2	135.0	134.5	133.5	133.0	132.0	131.0	130.0	129.0	128.0
	Alternate	138.9	142.1	145.4	148.5	151.1	154.3	157.0	159.7	162.5	165.3	168.2

Table 5: Shocks to Domestic Endogenous Variables for Baseline and Alternate Scenarios

Variable	Scenario	23Q2	23Q3	23Q4	24Q1	24Q2	24Q3	24Q4	25Q1	25Q2	25Q3	25Q4
Aggregate Demand	Baseline	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Alternate	-5.0	-4.0	-3.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Risk Prem	Baseline	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Alternate	7.0	6.0	5.0	4.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0
Food Inflation	Baseline	0.0	1.0	0.5	0.0	0.0	1.0	0.5	0.0	0.0	1.0	0.5
	Alternate	0.0	3.0	1.5	0.0	0.0	3.0	1.5	0.0	0.0	3.0	1.5
Exchange Rate	Baseline	0.0	2.0	1.0	0.0	0.0	2.0	1.0	0.0	0.0	2.0	1.0
	Alternate	0.0	4.0	2.0	0.0	0.0	4.0	2.0	0.0	0.0	4.0	2.0

Figure 13: Assumptions regarding External Environment for Baseline and Alternate Scenarios



6.3.2 Results of Scenario Analysis

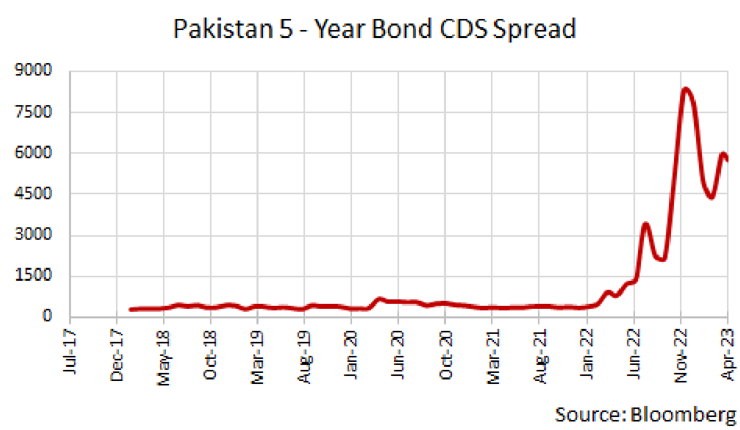
Results of scenario analysis are presented in Table 6 and Figure 15[18]. We produce out-of-sample baseline and alternate scenario forecasts over the horizon 23Q2 - 25Q4. To analyze how the changes in macroeconomic environment lead to changes in forecasts, we also present baseline forecast made at the end of 22Q4. Although there is difference of only one quarter between 22Q4 and 23Q1 yet, comparison of forecasts shows that heightened level of uncertainty led to considerable deterioration of outlook.

[18] Statistical uncertainty around baseline forecasts can be assessed through RMSEs presented in section 5.

23Q1 Baseline and Alternate Scenario Our baseline and alternate scenario forecasts are based upon our assumptions regarding external environment and realization of shocks, which are presented in Table 4 and Table 5, respectively. In baseline, headline (green dotted line in Figure 15), core and food inflation rates are expected to decline. GDP growth is likely to start recovery after observing negative quarterly YoY growth in 23Q1. Baseline forecast for FY24 average inflation is 16.78 percent and FY24 GDP growth is 5.96 percent. Alternate scenario, however, owing to assumed shocks to risk premium, food inflation, aggregate demand and adverse developments in external environment shows that headline inflation may persist at elevated level while core and food inflation rates may increase further. On the other hand, GDP growth may dip further in the negative territory. Alternate scenario FY24 average forecasts for headline inflation and GDP growth are 27.36 percent and 2.09 percent, respectively.

Comparison of Baseline Forecasts for 22Q4 vs. 23Q1 Major deterioration in economic indicators was observed in the form of increase in risk premium (Figure 14) and depreciation of local currency from quarterly average of Rs./$ 226 in 22Q4 to Rs./$ 284 in 23Q1. As shown in IRFs section, increase in risk premium leads to (Figure 7) exchange rate depreciation and inflation, necessitating an increase in interest rate which lowers output growth. Substantial depreciation has played a major role in deterioration of macroeconomic outlook during 23Q1.

Figure 14: Credit Default Swap Spread for Pakistan 5 - Year Bond



Comparison[19] of baseline forecasts obtained on the basis of data till 22Q4 and data till 23Q1 shows that headline inflation outlook deteriorates from 13.32 percent average inflation

[19] Assumptions regarding external environment and shocks to domestic variables are same for both baseline forecasts.

for FY24 to 16.78 percent for the same period. Average GDP growth for FY24 forecast improves from 4.50 percent to 5.96. Improvement in GDP growth forecast might be attributable to base effect and exchange rate depreciation that could boost export and GDP.

Table 6: Forecasts of Key Domestic Variables under Baseline and Alternate Scenarios

Variable	Scenario	22Q4	23Q1	23Q2	23Q3	23Q4	24Q1	24Q2	24Q3	24Q4	25Q1	25Q2	25Q3	25Q4	FY24	FY25
CPI Inf YoY	Baseline - 23Q1	22.20	27.40	24.93	19.99	19.36	14.05	13.71	13.09	12.28	11.40	10.63	10.00	9.49	16.78	11.85
	Baseline - 22Q4	22.20	22.38	20.02	15.20	13.85	12.70	11.53	10.39	9.55	9.00	8.76	8.67	8.59	13.32	9.42
	Alternate - 23Q1	22.20	27.40	27.13	25.59	28.65	27.04	28.17	27.63	25.82	23.09	19.99	16.93	14.16	27.36	24.13
Food Inf YoY	Baseline - 23Q1	29.40	37.30	35.25	29.90	25.64	15.41	11.41	8.84	6.78	5.23	4.21	3.58	3.22	20.59	6.26
	Baseline - 22Q4	29.40	28.31	23.67	16.58	11.18	8.54	6.69	5.45	4.72	4.48	4.54	4.65	4.65	10.75	4.80
	Alternate - 23Q1	29.40	37.30	38.85	39.28	41.05	36.36	33.74	29.92	25.01	19.86	15.28	11.68	9.14	37.61	22.52
Core Inf YoY	Baseline - 23Q1	15.00	13.30	12.13	11.20	11.03	13.99	15.80	16.51	16.42	15.85	15.09	14.33	13.67	13.00	15.97
	Baseline - 22Q4	15.00	16.40	17.13	17.08	16.82	16.14	15.10	13.95	12.87	12.05	11.53	11.28	11.20	16.29	12.60
	Alternate - 23Q1	15.00	13.30	12.91	13.52	15.47	21.04	25.03	27.20	27.58	26.38	23.99	20.95	17.75	18.77	26.29
GDP Growth YoY	Baseline - 23Q1	0.97	-1.27	1.49	4.64	5.88	7.93	5.40	4.24	3.97	3.87	3.92	4.09	4.32	5.96	4.00
	Baseline - 22Q4	0.97	1.47	2.58	4.79	5.42	4.07	3.72	3.60	3.71	3.97	4.28	4.57	4.77	4.50	3.89
	Alternate - 23Q1	0.97	-1.27	-3.63	-2.21	-0.77	3.35	7.97	9.64	9.60	7.47	5.30	4.29	4.00	2.09	8.00

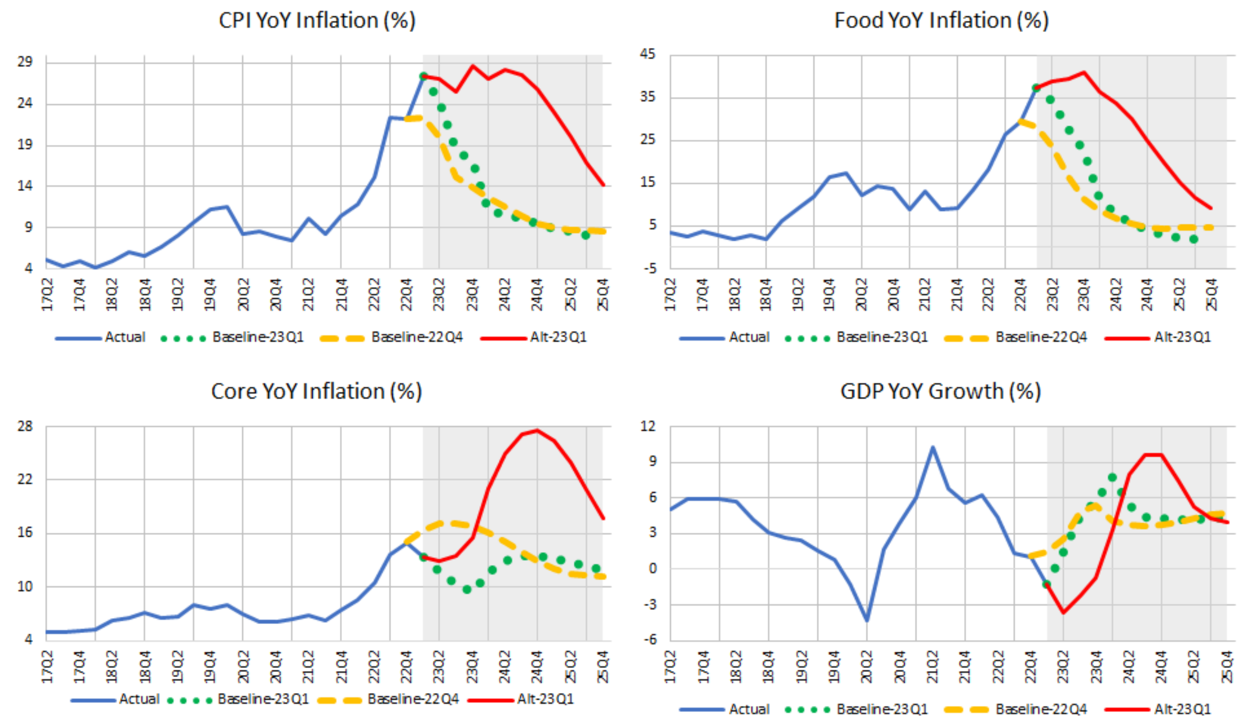
Table 7: Accommodative and Extinguishing Monetary Policy Experiments relative to Baseline

Variable	Scenario	23Q1	23Q2	23Q3	23Q4	24Q1	24Q2	24Q3	24Q4	25Q1	25Q2	25Q3	25Q4	FY24	FY25
CPI Inf YoY	Baseline	27.4	24.93	19.99	19.36	14.05	13.71	13.09	12.28	11.40	10.63	10.00	9.49	16.78	11.85
	Acc. MP	27.4	25.95	22.53	23.63	19.97	19.99	18.86	16.94	14.67	12.52	10.72	9.35	21.53	15.75
	Extngsh. MP	27.4	23.91	17.45	15.09	8.12	7.41	7.31	7.62	8.13	8.74	9.27	9.63	12.02	7.95
GDP Growth YoY	Baseline	-1.27	1.49	4.64	5.88	7.93	5.40	4.24	3.97	3.87	3.92	4.09	4.32	5.96	4.00
	Acc. MP	-1.27	2.00	5.63	7.26	9.51	6.33	4.37	3.33	2.67	2.56	2.84	3.34	7.18	3.23
	Extngsh. MP	-1.27	0.97	3.65	4.49	6.34	4.46	4.11	4.61	5.07	5.28	5.33	5.31	4.74	4.77

Table 8: Monetary and Fiscal Policy Mix Experiments relative to Baseline

Variable	Scenario	23Q1	23Q2	23Q3	23Q4	24Q1	24Q2	24Q3	24Q4	25Q1	25Q2	25Q3	25Q4	FY24	FY25
CPI Inf YoY	Baseline	27.4	24.93	19.99	19.36	14.05	13.71	13.09	12.28	11.40	10.63	10.00	9.49	16.78	11.85
	Acc. MP + FP	27.4	25.66	21.72	22.17	17.90	17.83	17.09	15.92	14.52	13.08	11.72	10.52	19.90	15.16
	Extngsh. MP + FP	27.4	23.62	16.64	13.64	6.04	5.24	5.54	6.60	7.98	9.30	10.27	10.80	10.39	7.35
GDP Growth YoY	Baseline	-1.27	1.49	4.64	5.88	7.93	5.40	4.24	3.97	3.87	3.92	4.09	4.32	5.96	4.00
	Acc. MP + FP	-1.27	-0.03	2.42	3.36	5.24	6.00	6.39	6.76	6.89	5.11	4.32	4.10	4.26	6.29
	Extngsh. MP + FP	-1.27	-1.06	0.44	0.59	2.07	4.14	6.12	8.04	9.30	7.82	6.81	6.07	1.81	7.82

Figure 15: Forecasts of Domestic Variables for Baseline and Alternate Scenarios

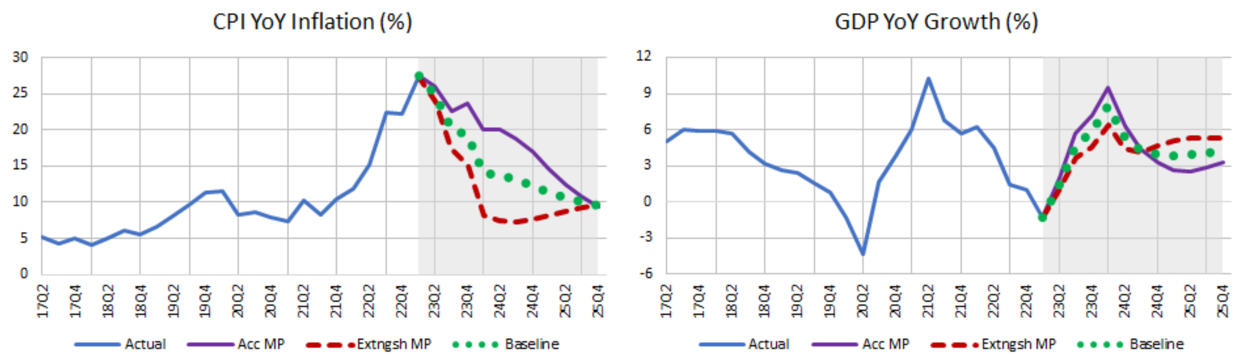


Monetary Policy Experiments Figure 16 and Table 7 show results of monetary policy experiments mentioned above. As expected, accommodative and extinguishing scenarios are respectively associated with higher and lower inflation rates relative to baseline. FY24 average inflation under accommodative and extinguishing scenarios is 21.53 percent and 12.02 percent, respectively. Intuitively, exchange rate depreciation is also higher in accommodative scenario. However, GDP growth shows a mixed pattern, accommodative scenario provides higher growth only till four quarters. In our monetary policy experiment, four quarters is the time by when interest rate is set through discretion rather than monetary policy rule[20]. After this time, elevated inflation calls for higher interest rate hike which pushes GDP growth lower than even the extinguishing scenario.

Monetary and Fiscal Policy Mix Experiments Figure 17 and Table 8 show the results of policy mix experiment. Under the policy mix experiment, FY24 average inflation under accommodative and extinguishing scenarios is 19.90 percent and 10.39 percent, respectively. FY24 average GDP growth for accommodating and extinguishing scenarios is

[20] We tried to fix the interest rate path for entire projections horizon. However, the model exhibits explosive behavior under such scenario.

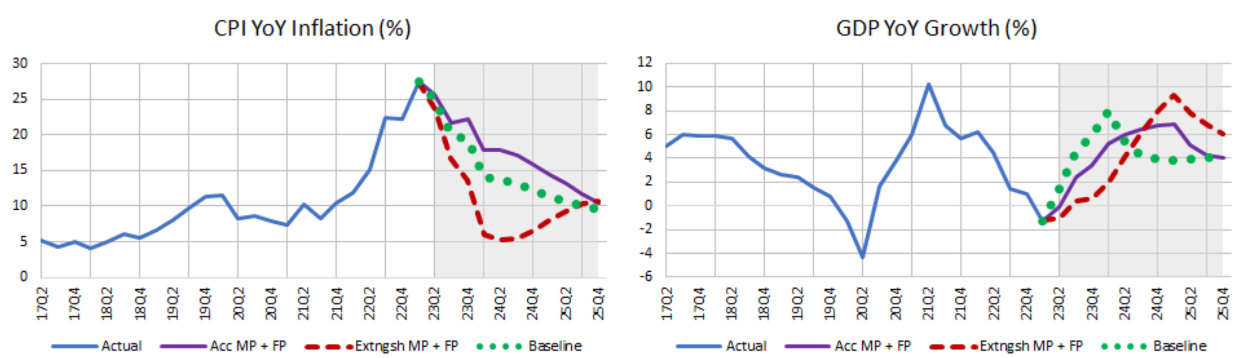
Figure 16: Accommodative and Extinguishing Monetary Policy Experiments relative to Baseline



4.26 percent and 1.81 percent respectively. However, growth pattern reverses in FY25 when extinguishing scenario dominates with 7.82 percent growth relative to 6.29 percent growth in the accommodative scenario. Exchange rate forecasts are almost unaffected by addition of fiscal policy.

In sum, we note that accommodative scenario leads to substantial increase in inflation which ultimately necessitates rise in interest rate. Owing to this rise in interest rate, initial spurt in GDP growth is more than eliminated. In current circumstances, accommodative scenario may lead to increase in macroeconomic volatility, without offering reasonable gains in terms of GDP growth.

Figure 17: Monetary and Fiscal Policy Mix Experiments relative to Baseline



7 Conclusion

We present estimated version of quarterly projections model that was actually operationalized for Pakistan by Ahmad & Pasha (2015). The model is a multi-sector extension of the model developed primarily for inflation targeting by Berg et al. (2006*a*) and Berg et al. (2006*b*). Berg et al. (2006*b*) suggest that Bayesian estimation of the calibrated model is a logical step in the strategy for improvement of analytical framework used for inflation targeting. Our estimation results show substantial changes in long run and short run parameters relative to initial calibration. Fresh estimates of long run parameters indicate that steady state inflation has increased slightly while steady state real exchange rate appreciation has decreased substantially.

In comparison with calibration offered by Ahmad & Pasha (2015), following key differences are found. First, real interest rate gap is far more important in shaping monetary conditions. Second, core inflation is more persistent and more sensitive to real marginal costs. Third, domestic food inflation is more sensitive to external factors like exchange rate and world food prices. Fourth, pass through of world oil prices to local energy inflation has decreased. Fifth, monetary policy response to exchange rate depreciation is quite low. Sixth, central bank's interest rate rule gives almost same weight to expected inflation and output gap. Finally, substantial differences in persistence parameters of shock processes and standard deviations are observed.

Pseudo out-of-sample forecast performance comparison shows that B-ML specification outperforms purely data driven ML estimates and calibration in case of inflation, GDP growth rate, interest rate and exchange rate. Substantial gains in forecast performance are observed in case of real GDP growth rate and interest rate.

We utilize the model for gap analysis and scenario analysis. Results of gap analysis show that currently, output gap is negative, implying recession. Real exchange rate gap is positive, implying under valuation of domestic currency. In scenario analysis, we incorporate risk factors like political instability, climate risks, commodity prices and global financial conditions to show that simultaneous materialization of risk factors may lead to substantially higher inflation coupled with lower GDP growth. Monetary and fiscal policy experiments show that under current circumstances, expansionary policy stance may add to macroeconomic volatility and is likely to be unsustainable in the medium term.

Going forward, the framework can be improved by explicit modeling of fiscal and financial sectors. Our alternate scenario projections may be used for macro stress testing exercise as the scenario offers theoretically consistent trajectories of macro variables under hypothetical stressed environment while fulfilling requirements of extremeness and plausibility.

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Appendix A Key Equations Included in the Model

Aggregate Demand

Dynamic IS curve

$$\hat{y}_t = a_1 \hat{y}_{t-1} - a_2 mci_{t-1} + a_3 \hat{y}_t^* + \varepsilon_t^y \tag{6}$$

Monetary conditions index

$$mci_t = a_4 \left(\hat{r}_t\right) + \left(1 - a_4\right)\left(-\hat{z}_t\right) \tag{7}$$

Aggregate Supply

NKPC for core inflation

$$\pi_t^{core} = b_1 \pi_{t-1}^{core} + \left(1 - b_1\right) E_t \pi_{t+1} + b_2 rmc_t^{core} + \varepsilon_t^{\pi^{core}} \tag{8}$$

Real marginal costs for core inflation

$$rmc_t^{core} = b_3 \hat{y}_t + \left(1 - b_3\right) \hat{z}_t \tag{9}$$

NKPC for food inflation

$$\pi_t^{food} = b_{21} \pi_{t-1}^{food} + \left(1 - b_{21}\right) E_t \pi_{t+1} + b_{22} rmc_t^{food} + \varepsilon_t^{\pi^{food}} \tag{10}$$

Real marginal costs for food inflation

$$rmc_t^{food} = b_{23} \hat{z}_t^{food} + \left(1 - b_{23}\right) \hat{y}_t \tag{11}$$

Relative price for food

$$\hat{z}_t^{food} = P_{world}^{food} + \hat{z}_t - P_t^{food} \tag{12}$$

Energy price NKPC

$$\pi_t^{oil} = b_{31} \pi_{t-1}^{oil} + \left(1 - b_{31} - b_{32}\right) E_t \pi_{t+1} + b_{32} rmc_t^{oil} + \varepsilon_t^{\pi^{oil}} \tag{13}$$

Real marginal costs for energy inflation

$$rmc_t^{oil} = \Delta p_t^{oil} + \Delta s_t^{USD} - \Delta \bar{z}_t \tag{14}$$

Headline inflation

$$\pi_t = w^{oil}\pi_t^{oil} + w^{food}\pi_t^{food} + \left(1 - w^{oil} - w^{food}\right)\pi_t^{core} + \varepsilon_t^{\pi} \tag{15}$$

External Sector

Modified UIP

$$s_t = (1 - e_1) E_t s_{t+1} + e_1 \left[s_{t-1} + \frac{2}{4}\left(\bar{\pi}_t - \bar{\pi}_t^* + \Delta\bar{z}_t\right)\right] + \frac{(i_t^* - i_t + prem_t)}{4} + \varepsilon_t^s \tag{16}$$

Central Bank's Policy Rule

$$\begin{aligned} i_t &= g_1 \left[4(s_{t+1} - s_t) + i_t^* + prem_t\right] + \\ &\quad (1 - g_1)\left[f_1 i_{t-1} + (1 - f_1)\left\{i_t^n + f_2\left(E_t\pi_{t+4} - \pi_t^T\right) + f_3\hat{y}_t\right\}\right] + \varepsilon_t^i \end{aligned} \tag{17}$$

Appendix B Maximum Likelihood Estimation Results

Table 9: Comparison of ML Estimates with Calibration and B-MLE

Parameter	**Symbol**	**Calibration**	**ML**	**Estimated**
Core inflation persistence	b_1	0.70	0.57	0.76
Impact of RMC on core inflation	b_2	0.70	0.41	0.38
Weight of outputgap in RMC	b_3	0.50	0.79	0.70
Food inflation persistence	b_{21}	0.50	0.08	0.25
Impact of food RMC on food inflation	b_{22}	0.40	0.72	0.45
Energy inflation persistence	b_{31}	0.10	-0.05	0.05
Impact of energy RMC on domestic energy inflation	b_{32}	0.40	0.07	0.20
Interest rate reaction to inflation forecast deviation	f_2	2.00	12.12	1.84
Interest rate reaction to output gap	f_3	1.00	4.28	1.83
Persistence of shock to trend real GDP growth rate	$\rho^{\epsilon^{\bar{y}}}$	0.50	0.81	0.54
Persistence of shock to foreign GDP gap	$\rho^{\epsilon^{\hat{y}^*}}$	0.50	0.77	0.67
Persistence of shock to foreign interest rate	$\rho^{\epsilon^{rn^*}}$	0.50	0.94	0.91
Persistence of shock to foreign inflation	$\rho^{\epsilon^{\pi^*}}$	0.50	0.44	0.41
Persistence of shock to world food inflation	$\rho^{\epsilon^{\pi^{wfood}}}$	0.10	0.24	0.45
Persistence of shock to world oil inflation	$\rho^{\epsilon^{\pi^{woil}}}$	0.10	0.19	0.30
Std dev of shock to headline inflation	σ^{π}	0.20	0.20	0.26
Std dev of shock to core inflation	$\sigma^{\pi^{core}}$	0.50	0.32	0.44
Std dev of shock to energy inflation	$\sigma^{\pi^{oil}}$	3.00	3.49	4.77
Std dev of shock to food inflation	$\sigma^{\pi^{food}}$	1.20	1.24	1.35
Std dev of shock to foreign inflation	σ^{π^*}	0.60	0.57	0.71
Std dev of shock to trend real GDP growth	$\sigma^{\bar{y}}$	0.40	0.19	0.76
Std dev of shock to world food price	$\sigma^{\pi^{wfood}}$	3.00	5.71	5.00

Figure 18: Log-Likelihood in neighbourhood of ML estimates - 1

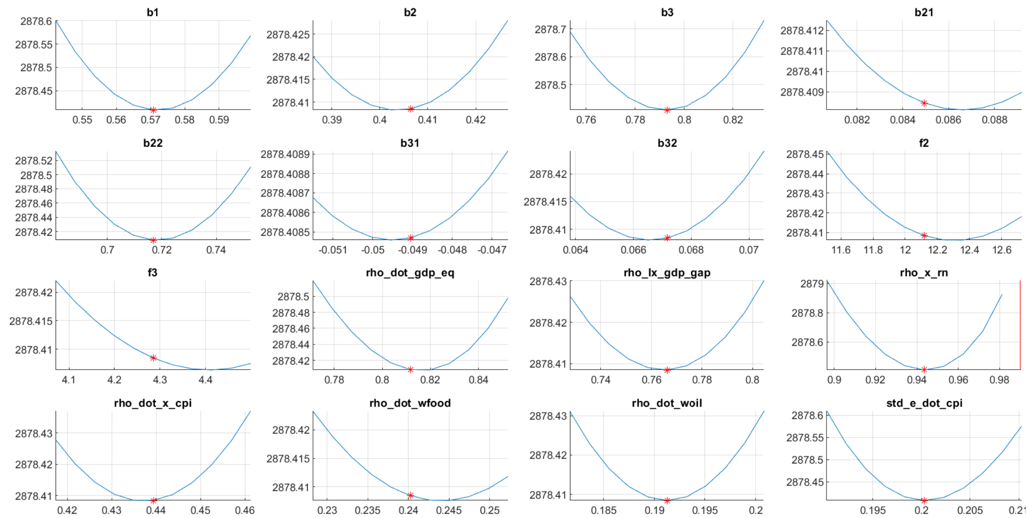
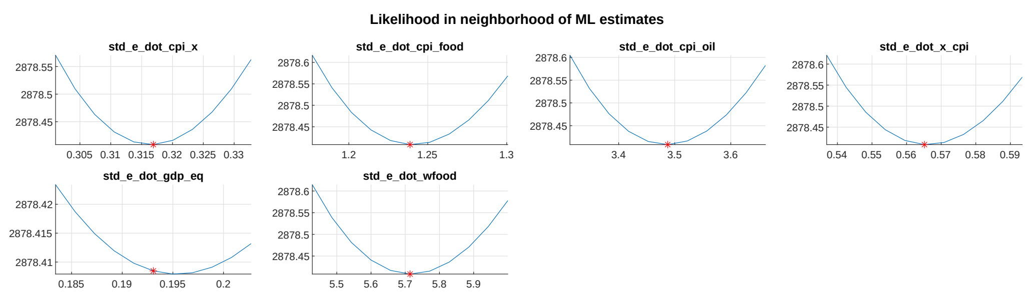


Figure 19: Log-Likelihood in neighbourhood of ML estimates - 2



Appendix C Prior and Posterior Distributions

Figure 20: Posterior distributions - 1

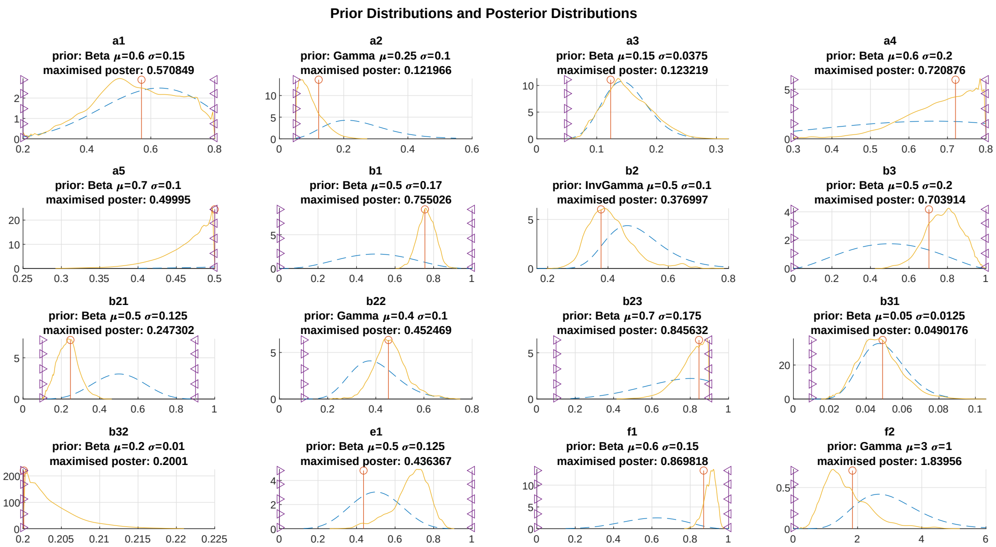


Figure 21: Posterior distributions - 2

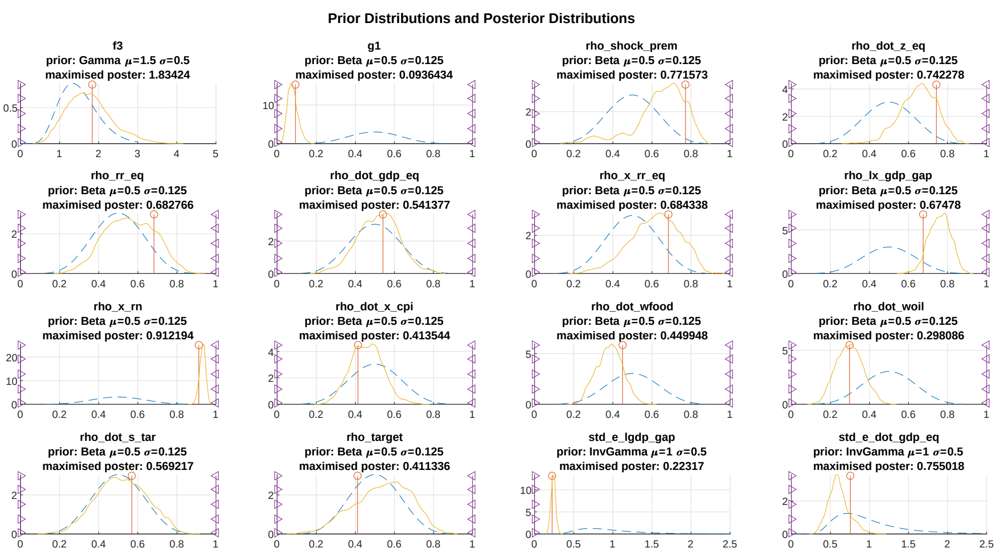


Figure 22: Posterior distributions - 3

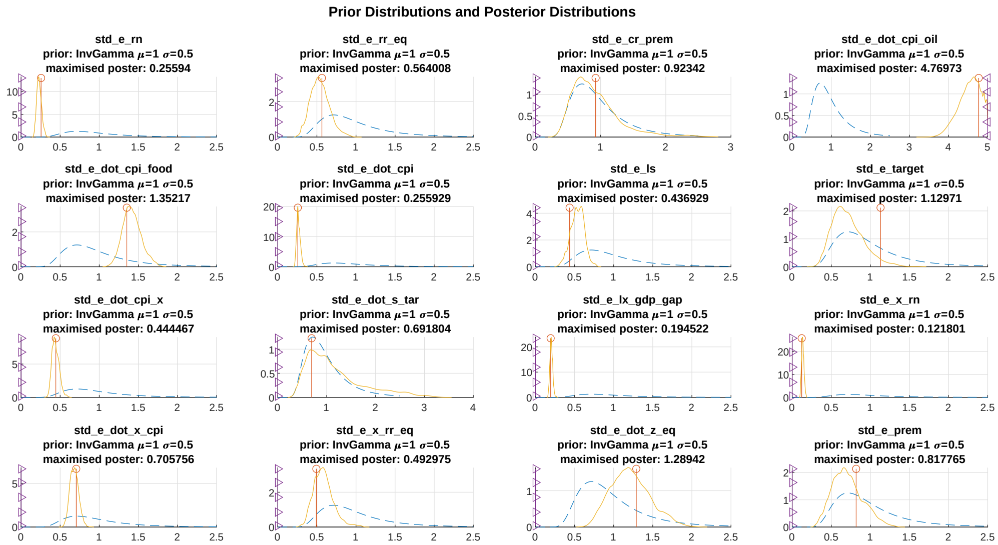


Figure 23: Posterior distributions - 4